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Abstract

Suppose a firm uses two inputs that are substitutes to generate output according to a smooth production function. Each input is supplied by a single contractor. The firm would like to make both suppliers compete with one another. However, since inputs are imperfect substitutes, it cannot resort to winner-take-all competition. We allow the firm to use a modified first-price auction. The procurer announces a demand functions for each input, each one of them depending on both suppliers' unit price bids. Given those demands, contractors simultaneously bid unit prices and sell according to announced input demands. We show that the procurer has an incentive to announce demands that do not reflect its true production technology. The firm overstates input substitutability and understates its own willingness to pay so as to make the auction more competitive.

Keywords: Procurement, Auctions, Mechanism Design.
JEL Classifications: C72, D44

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1 Introduction

Suppose a firm uses two inputs that are substitutes to generate output according to a smooth production function. Each input is supplied by a single contractor subject to linear cost functions. Contractors' unit costs are their private information. The firm would like to make both suppliers compete with one another, since the inputs they provide are substitutes. However, since they are imperfect substitutes, it would rather not resort to any form of winner-take-all competition.

We examine here a context where the firm procures those inputs using a modified version of a first-price, reverse auction. In this auction, the procurer announces a demand functions for each input, each one of them depending on both unit price bids. Given the stipulated demands, contractors simultaneously bid unit prices that determine how many units each of them sells at their chosen prices.

The degree of competition induced by the auction, and therefore its profitability for the procurer, depend upon the degree of substitutability between the two inputs and the procurer's maximum willingness to pay, as reflected in the stipulated demand functions. If the degree of substitutability is low, or the maximum willingness to pay is high, contractors will choose high bids -ie. high unit prices- that translate into a low profit for the procurer. However, the procurer may strategically engineer the demand functions, by announcing demands that exhibit a higher degree of substitutability, or a lower maximum willingness to pay, than the true values that follow from its actual technology. Strategically engineered demand functions can contribute to increase the procurer's expected profit by inducing contractors to compete more fiercely. However, they may also distort the input mix. Therefore, the optimal degree of strategic engineering or fine-tuning must trade off its benefits and costs.

In this note, we show that the firm may indeed want to announce demand functions that do not reflect its own technology. Specifically, we select a technology that generates linear actual input demands. Then, we allow the firm to announce demand functions with parameters that do not correspond to their actual values, so as to enhance its own profitabiltiy, within the same class of functions its actual demands belong to. We examine two possible cases. First, when supplier costs are perfectly correlated, we show that by extremely overstating substitutability, the firm may procure the efficient input levels and extract all surplus at the same time. Then, when supplier costs are i.i.d, we obtain a less extreme result. The firm optimally overstates substitutability and understates its own willingness to pay so as to make the auction more competitive.

Although the use of auctions in procuring inputs that are substitutes has, to our knowledge, not been tackled in the literature, there are somewhat related contributions on multiple-sourcing (see, for example, Anton and Yao, 1989). However, in that literature, the procurer has a preference for procuring from multiple contractors because he is concerned with giving incentives for procurement-related R&D and with assuring future competition among suppliers.[1] Here, procuring from multiple suppliers stems from the simple fact that there is imperfect input substitutability in the firms' production technology.

2 The Model

A firm purchases two inputs from two independent contractors. Let q_i be the quantity of input i bought from the corresponding supplier, $i = 1, 2$. Inputs are imperfect substitutes. To ensure tractability, the firm's revenue function, R, is assumed quadratic,

$$R(q_i, q_j) = \alpha(q_i + q_j) - \frac{\beta}{2(1+\sigma)}(q_i^2 + q_j^2 + 2\sigma q_i q_j). \quad (1)$$

Thus, profit maximization yields "true" input demand functions, $Q_i^t(p_1, p_2)$, that are linear.

$$Q_i^t(p_1, p_2) = \frac{\alpha}{\beta} - \frac{1}{\beta(1-\sigma)}\left(p_i - \sigma p_j\right). \quad (2)$$

Here, α represents the firm's maximum willingness to pay for each input, and $\frac{1}{\beta}$ the size of the market. The parameter σ reflects input substitutability, and ranges from $\sigma = 0$, when inputs are independent, to σ approaching 1, when they become perfect substitutes.

The assumed revenue and the associated *true* demand functions are borrowed from Shubik and Levitan (1980) who introduced them in the context of Bertrand market games with differentiated products to remedy an undesirable feature of the often used Bowley specification (Bowley, 1924; Singh and Vives, 1984), in which an increase in the measure of substitutability reduces the size of the market and makes it shrink away completely as products become perfect substitutes.[2]

Contractors are subject to linear cost functions. Their unit costs x_i are their private information. We will examine below two different cases, in which x_1 and x_2 are either (i) perfectly correlated, or (ii) i.i.d. according to a given, continuous c.d.f. F, with support $[c, d]$, where $0 \leq c < d < \alpha$.

Inputs are procured by a (fine-tuned) first-price reverse auction. The procurer announces demand functions, $Q_i(p_i, p_j)$. These have the same functional form as the true demands in (2). Still, the procurer need not choose the true parameter values (α, β, σ), but may select alternative values (a, b, s). Given announced demands, the firm asks contractors to simultaneously bid unit prices, p_1, p_2 . After bids are submitted, they are revealed, and then contractors must deliver the quantities, $Q_i(p_i, p_j)$ in exchange for payments $p_i Q_i(p_i, p_j)$.

In what follows, we describe the optimal choice of announced parameter values, (a, b, s), when the procurer wants to maximize his own profit, given by

$$R(Q_1(p_1, p_2), Q_2(p_1, p_2)) - p_1 Q_1(p_1, p_2) - p_2 Q_2(p_1, p_2),$$

[1] For a critical assessment of the benefits of second-sourcing, see Riordan and Sappington (1989).

[2] We express these demand functions in the slightly more convenient yet equivalent form, introduced by Collie and Le (2015).

starting with the case where supplier unit costs are perfectly correlated.

3 Correlated costs and full surplus extraction

Suppose unit costs are perfectly correlated and this fact is common knowledge. Then, each supplier knows not only his unit cost but also that of his competitor, since $x_1 = x_2 = x$. The procurer, though, does not know x and still considers it a random variable. We will now show that, for any true parameters (α, β, σ), the procurer can announce parameters (a, b, s) such that he extracts all surplus from suppliers while maximizing his own revenue.

In this case, consider the family of possible announced demand functions:

$$Q_i(p_i, p_j) = \begin{cases} \frac{a}{b} - \frac{1}{b(1-s)}(p_i - sp_j) & \text{if } s \in [0,1) \\ \frac{2}{b}(a-p) & \text{if } s = 1 \text{ and } p_i < p_j \\ 0 & \text{if } s = 1 \text{ and } p_i > p_j \\ \frac{1}{b}(a-p) & \text{if } s = 1 \text{ and } p_i = p_j. \end{cases} \tag{3}$$

Based on these stipulated demand functions, the equilibrium bid functions are, for $i \in \{1, 2\}$:

$$p_i(x) = \arg\max_p (p - x) Q_i(p, p(x)) \Rightarrow p_i(x) = \frac{a(1-s) + x}{2-s}, \tag{4}$$

which yields $p_i(x) = x$, if $s = 1$, and we conclude:

Proposition 1 (Perfectly correlated unit cost) *If unit costs are perfectly correlated, the procurer's expected profit is maximized if he treats the two inputs as perfect substitutes, setting $s = 1$, and announcing demand functions with $a = \alpha, b = \beta$. The procurer extracts the maximum surplus.*

Proof. By setting $s = 1$, $a = \alpha$ and $b = \beta$, equilibrium bids equal the unit cost x, so suppliers earn zero profits. The profit-maximizing, symmetric input demand will thus be

$$q^*(x) = \arg\max_q R(q, q) - 2qx$$

which yields $q^*(x) = \frac{\alpha - x}{\beta}$. It follows that $Q_i(p_1(x), p_2(x)) = Q_i(x, x)$, $i = 1, 2$. The procurer implements the surplus-maximizing allocation and extracts all surplus. ■

This simple mechanism extracts the maximal surplus, regardless of the size of the true substitution parameter σ (particularly, even if goods are independent). Given the well-known results in Crémer and McLean (1988), it is not surprising that one can find mechanisms that extract all surplus if costs are correlated. However, it may be surprising that a simple, belief-free, mechanism can achieve this outcome, though it relies on correlation being perfect. Essentially, for any possible cost level and for any true value of σ,the procurer makes both suppliers compete à la Bertrand as if they provided homogeneous inputs.

4 The case of independent private values

Suppose now that his unit cost is each supplier's private information, and that unit costs are i.i.d. according to a the continuous c.d.f. F, with support $[c, d]$, where $0 \leq c < d < \alpha$.

Now using an auction cannot possibly achieve full surplus extraction, since prices will not be driven down to unit cost levels. In addition, setting $s = 1$ and stipulating the demand functions as in (3) is not desirable, since in that case only one good will be procured, which forgoes the benefit of using both inputs. However, as we will show, the procurer will strategically announce demand functions with $s > \sigma$ and $a < \alpha$. Then, he induces bidders to behave more aggressively by overstating substitutability and understating his own maximum willingness to pay for inputs.

Suppose the procurer announces demand functions

$$Q_i(p_i, p_j) := \frac{a}{b} - \frac{1}{b(1-s)}\left(p_i - s p_j\right),$$

with $a, b > 0$ and $s \in [0, 1)$. Bid functions must now be Bayesian Nash equilibrium strategies. Then, equilibrium bidding functions $(p_1(x_1), p_2(x_2))$ satisfy

$$p_i(x_i) = \arg\max_p (p - x_i) \int_c^d Q_i(p, p_j(x_j)) dF(x_i) \quad i, j = 1, 2, i \neq j.$$

Solving supplier i's problem, its best response is

$$p_i(x_i) = \frac{1}{2}[a(1-s) + sE(p_j(x_j)) + x_i],$$

which yields Bayesian Nash equilibrium strategies

$$p_i(x_i) = \frac{2a(1-s) + (2-s)x_i + s\bar{x}}{2(2-s)}, \quad i = 1, 2,$$

where $\bar{x}$ denotes the expected value of a firms' unit cost.

The announced market size parameter, b, does not affect bidding. Therefore, the procurer has no incentive to set $b \neq \beta$, and for convenience we normalize $b = \beta = 1$.

Given equilibrium strategies, the associated equilibrium quantities supplied are[3]

$$\begin{aligned} Q_i^e(\mathbf{x}; a, s) &= Q_i(p_i(x_i), p_j(x_j)) \\ &= \frac{2a(1-s) - (2-s)(x_i - s x_j) - (1-s)\overline{x}}{2(2-s)(1-s)}, \end{aligned} \quad (5)$$

[3] Note that if x_i is significantly larger than x_j, it may be the case, for some values of a, s, that $Q_i^e(\mathbf{x}; a, s)$ becomes negative. Here, we proceed as if equilibrium quantities were necessarily positive. We will later check that there is a parameter region where quantities are indeed positive.

where $\mathbf{x} = (x_1, x_2)$. The procurer's profit, as a function of suppliers' unit costs and announced parameters, is

$$\begin{aligned}\pi_0(\mathbf{x}; a, s) &= R(Q_1^e(\mathbf{x}; a, s), Q_2^e(\mathbf{x}; a, s)) - p_1(x_1)Q_1^e(\mathbf{x}; a, s) - p_2(x_2)Q_2^e(\mathbf{x}; a, s) \\ &= \alpha \sum_{i=1}^{2} Q_i^e(\mathbf{x}; a, s) - \frac{1}{2(1+\sigma)} \left(\sum_i Q_i^e(\mathbf{x}; a, s)^2 + 2Q_1^e(\mathbf{x}; a, s)Q_2^e(\mathbf{x}; a, s) \right) \\ &\quad - \sum_{i=1}^{2} p_i(x_i)Q_i^e(\mathbf{x}; a, s)\end{aligned}$$

The procurer should choose (a, s) to maximize $E(\pi_0(\mathbf{x}; a, s))$. That is, he should solve

$$\max_{a,s} \int_c^d \int_c^d \pi_0(\mathbf{x}; a, s) dF(x_1) dF(x_2).$$

We can now show that the procurer has an incentive to strategically set the parameters a and s at values that are different from the true ones.

Proposition 2 *The procurer can increase his expected payoff by strategically setting a higher than true substitution parameter, $s > \sigma$, and a lower than true willingness to pay parameter $a < \alpha$.*

Proof. The procurer's problem first-order conditions are[4]

$$\frac{2\alpha(2-s) - 2a(3-2s) + \overline{x}(1-s)}{(2-s)^2} = 0 \quad (6)$$

$$\frac{1}{2} \left[\frac{-4a^2(1-s) + 2s\overline{x}^2 + a(2-s)(4\alpha - 2\overline{x}) + 2\overline{x}(2-3s)}{(2-s)^3} - \frac{2(s-\sigma)var(x_i)}{(1+\sigma)(1-s)^3} + \frac{(2\overline{x} - 4\alpha)\overline{x}}{(2-s)^2} \right] = 0 \quad (7)$$

From (6), it follows that

$$a = \frac{\alpha(2-s) + \overline{x}(1-s)}{3-2s} \quad (8)$$

Consequently,

$$a - \alpha = \frac{\alpha(2-s) + \overline{x}(1-s)}{3-2s} - \alpha = -\frac{(1-s)(\alpha - \overline{x})}{3-2s} < 0.$$

Hence, the procurer underestimates his willingness-to-pay parameter. To show that he overestimates substitutability, let us examine (7) replacing a according to the expression obtained in (8). After that substitution, (7) becomes

$$\frac{3(\alpha - \overline{x})^2}{(3-2s)^2} - \frac{2(s-\sigma)var(x_i)}{(1+\sigma)(1-s)^3} = 0 \quad (9)$$

[4]Note that $y := x_1 - x_2$ gives $E(y^2) = E(y)^2 + var(y) = 2var(x)$, using the fact that $x := x_1 = x_2$ are i.i.d. random variables.

It is straighforward to check that the LHS in (9) is positive if $s \leq \sigma$. Furthermore, as $\sigma < 1$, that LHS goes to $-\infty$ when $s \to 1$. It then follows that (9) will be satisfied for some $s > \sigma$. ■

The procurer will then induce stronger competition between suppliers by announcing a demand system that reflects more substitutabilty, and less overall willingness to pay, than would follow from his actual production technology.

Example 3 *Assume that x_1, x_2 are uniformly distributed on the unit interval, and the procurer's technology is such that $\alpha = 10$, $\sigma = 0.4$. In this case, it follows from (8) that the procurer will set $a = 7.36$. As for s, Figure 1 below depicts the LHS of (7) when a is selected optimally (i.e. when (6) holds) as a function of s. Clearly, the procurer chooses $s > \sigma = 0.4$.*

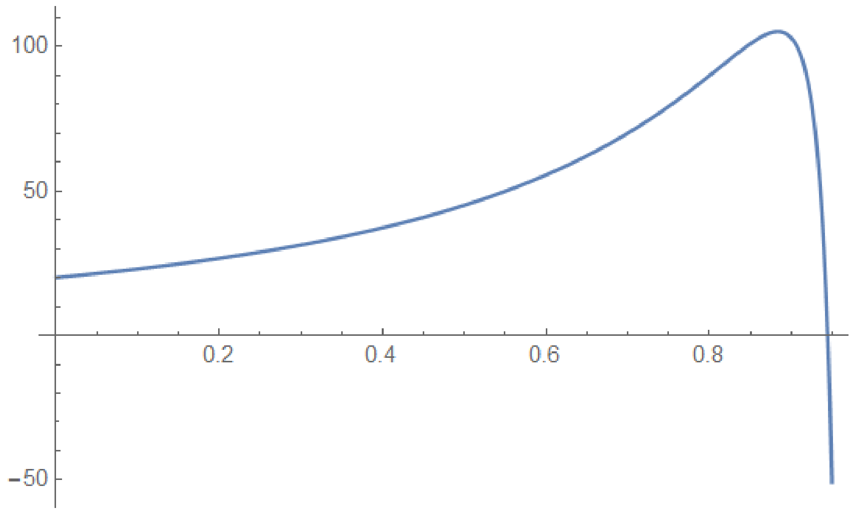

Figure 1

The results described so far have been derived under the assumption that equilibrium quantities are positive for any combination of supplier unit costs. However, as is clear by examining the expression for equilibrium quantities in (5), that need not be the case. At least for some values of a and s, if a supplier is significantly less efficient than its rival, the LHS of (5) may become negative, which is absurd. However, for any distribution F, we can always find a parameter region (that is, a set of values for α, σ) such that the procurer will select optimal values of a and s that yield positive equilibrium quantities with probability one. Figure 2 below shows the parameter region where ensuring that equilibrium quantities is not an issue for the case described in Example 3.

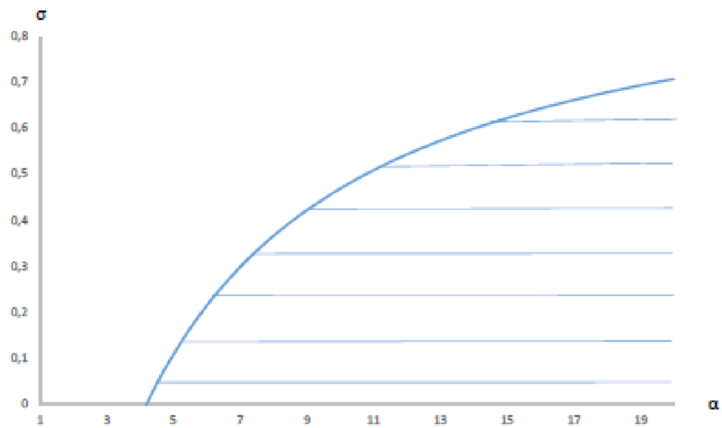

Figure 2

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