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Bera, Anil
Montes-Rojas, Gabriel
Sosa Escudero, Walter

General tests for nonlinear restrictions under local misspecification

Anil K. Bera* — University of Illinois
Gabriel Montes-Rojas† — IIEP - BAIRES - CONICET
Walter Sosa-Escudero‡ — Universidad de San Andrés - CONICET

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Abstract

This paper proposes a GMM based test for non-linear hypothesis that is robust to locally misspecified possibly non-linear hypothesis. That is, the procedure is based on an initial consistent GMM estimator of the nuisance parameters under a given set of possibly nonlinear restrictions. The new test for one particular non-linear hypothesis is consistent and has correct asymptotic size independently of whether the other also non-linear hypothesis are either correct or locally misspecified. As an illustration we apply them to the study of the rational expectations (RE) and neutrality (NE) hypotheses. Contrary to previous findings, our alternative robust LM-type test estimates the model under both RE and NE, and rejects the RE hypothesis.

*Department of Economics, University of Illinois, Urbana-Champaign, 1407 W. Gregory Drive, Urbana, IL 61801. Phone: +1 (217) 333-4596; Fax: +1 (217) 244-6678; e-mail: abera@illinois.edu

†Corresponding author: Instituto Interdisciplinario de Economía Política de Buenos Aires and CONICET, Facultad de Ciencias Económicas, Universidad de Buenos Aires, Córdoba 2122 2do piso, C1120AAQ, Ciudad Autónoma de Buenos Aires, Argentina, email: gabriel.montes@fce.edu.ar

‡Department of Economics, Universidad de San Andrés and CONICET, Vito Dumas 284, B1644BID, Victoria, Buenos Aires, Argentina, email: wsosa@udesa.edu.ar

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1 Introduction

Many relevant economic relationships imply non-linear restrictions on econometric models. A relevant examples are the classic papers by Mishkin (1982a, 1982b) where rational expectations and money neutrality impose non-linear restrictions on an otherwise linear model of GDP (or unemployment) and an aggregate demand policy variable.

When the null model is easier to handle the Rao's score or Lagrange Multiplier (LM) test is particularly convenient. A problematic situation arises when the hypotheses under study interact, in the sense that a test for any of them is affected by the validity of the others. Without loss of generality assume that there are only two competing hypothesis. Bera and Yoon (1993, BY hereafter) proposed a modified LM test LM for one hypothesis that is unaffected by the validity of the other one. The BY test requires that: a) both competing hypothesis are linear, b) the hypothesis not being tested for is either valid or invalid in a local sense, c) a fully parametric, likelihood model is specified. The BY principle has been successfully implemented; some examples include Anselin, Bera, Florax and Yoon (1996), Godfrey and Veall (2000), Bera, Sosa-Escudero and Yoon (2001), Baltagi and Li (2001), Baltagi, Song and Jung (2002) and Montes-Rojas (2010,2011).

Bera, Montes-Rojas and Sosa-Escudero (2010) extended the BY principle to the generalized method of moments (GMM) framework. More recently, Chernozhukov, Escanciano, Ichimura and Newey (2016) construct locally robust semiparametric GMM estimators inspired on the BY principle.

This paper proposes a GMM based test for non-linear hypothesis that is robust to locally misspecified possibly non-linear hypothesis. That is, the procedure is based on an initial consistent GMM estimator of the nuisance parameters under a given set of possibly nonlinear restrictions. The new test for one particular non-linear hypothesis is consistent and has correct asymptotic size independently of whether the other also non-linear hypothesis are either correct or locally misspecified.

In spite of the well known asymptotic equivalence of LM, Wald or likelihood ratio tests, LM based tests are particularly convenient for the problem under study. First, LM tests are usually favored when the restrictions under the null hypothesis imply a practically convenient model. Second, LM tests are invariant to the form of the nonlinear restrictions whereas Wald type tests are not. As noted by Gregory and Veall (1985b) alternative algebraically equivalent configurations of the same null hypothesis may affect the performance of Wald test significantly in small samples. Phillips and Park (1988) study this phenomenon using asymptotic analysis. Finally, there is not an obvious framework to robustify Wald type tests to the presence of misspecified alternatives, a subject that, as mentioned previously, has been already explored in the LM and GMM context.

As an illustration we apply them to the study of the rational expectations

(RE) and neutrality (NE) hypotheses. Mishkin (1982a,1982b), and later Bohara (1991) among others, implemented Wald tests for the joint hypothesis of RE and NE, and then marginal tests for each hypothesis assuming the other one does not hold. They find that RE cannot be rejected and strong evidence against NE. Gregory and Veall (1987) point out that Wald for RE are extremely sensitive in small samples to the way in which the non-linear restrictions in such models are parameterized (alternative tests are provided by Hoffman and Schmidt (1981) and Gregory and Veall (1985a)). Interestingly, and contrary to previous findings, our alternative robust LM-type test estimates the model under both RE and NE, and rejects the RE hypothesis.

The rest of the paper is organized as follows. Section 2 presents the statistical framework and the assumptions on the data and parameters. Section **??** develops the robust LM tests for liner and nonlinear restrictions. Section 4 applies the tests to the macro rational expectations hypothesis. Section 5 concludes and discuss further research ideas.

2 Non-linear restrictions and local misspecification

Consider a set of m population moment conditions that will be used to construct GMM estimators (see, e.g., Hansen (1982), Hansen and Singleton (1982) and Newey and West (1987)),

$$E\left[g(z,\theta_0)\right] = 0, \tag{1}$$

where $g(z,\theta)$ is an $m \times 1$ vector of functions of data and parameters, z is a $k \times 1$ random vector and θ is a $p \times 1$ vector of parameters.

When conditions (1) hold the sample moments

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} g(z_t,\theta),$$

should be close to zero when evaluated at $\theta = \theta_0$.

Let $\Omega_T(\theta)$ be an $m \times m$ positive semi-definite matrix. Define the loss function $Q_T(\theta) = -\frac{1}{2} g_T(\theta)^\top \Omega_T^{-1}(\theta) g_T(\theta)$. For asymptotic efficiency and to simplify the analysis we will assume $\lim_{T\to\infty} \Omega_T(\theta) = \lim_{T\to\infty} Var[\sqrt{T} g_T(\theta)]^{-1} = \Omega(\theta)$ and $\Omega = \Omega(\theta_0)$ (see Hansen (1982) and Newey and McFadden (1994)).[1]

Let $\nabla_\theta g(z,\theta) = \partial g(z,\theta)/\partial\theta^\top$ be the $m \times p$ Jacobian matrix of $g(z,\theta)$, $G(\theta) = E[\nabla_\theta g(z,\theta)]$ and $G_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} \nabla_\theta g(z_t,\theta)$. Define the counterpart of the (pseudo) score as $q_T(\theta) = -G_T(\theta)\Omega_T^{-1}(\theta) g_T(\theta)$, and $q_{j,T}(\theta)$ the $p_j \times 1$ subvector. Also, let $B(\theta) = G(\theta)^\top \Omega^{-1}(\theta) G(\theta)$, $B_T(\theta) = G_T(\theta)^\top \Omega_T^{-1}(\theta) G_T(\theta)$ and $B = B(\theta_0)$.

Consider two potentially nonlinear sets of $r_1 + r_2 = r$ restrictions,

[1]This can be extended to the continuous updating estimator of Hansen, Heaton and Yaron (1996). For instance, by a two-step GMM procedure where $\Omega_T = \frac{1}{T}(\sum_{t=1}^{T} g(z_t,\tilde\theta_T) g(z_t,\tilde\theta_T)^\top)^{-1}$ and $\tilde\theta_T$ is any first-step $\sqrt{T}$-consistent estimator of θ_0 (Newey and McFadden, 1994, p.2217).

$$a(\theta_0) = \begin{bmatrix} a_1(\theta_0) \\ a_2(\theta_0) \end{bmatrix} = \begin{bmatrix} 0_{r_1} \\ 0_{r_2} \end{bmatrix}.$$

Define $A(\theta) = \nabla_\theta a(\theta)$ and $A = A(\theta_0)$, an $r \times p$ matrix of rank r and consider the partition $A(\theta) = [\nabla_\theta a_1(\theta)^\top \; \nabla_\theta a_2(\theta)^\top]^\top = [A_1(\theta)^\top \; A_2(\theta)^\top]^\top$.

We are interested in the null hypothesis $H_0^1 : a_1(\theta_0) = 0$ against the local alternative $H_A^1 : a_1(\theta_0) = d_1/\sqrt{T}$. The validity of a test for H_0^1 will in general depend on a second restriction, $H_0^2 : a_2(\theta_0) = 0$. When $H_A^2 : a_2(\theta_0) = d_2/\sqrt{T}$ holds the model is *locally misspecified* in the sense that a hypothesis that is not under scrutiny is false in a local sense. For completeness, define the joint null hypothesis $H_0^{12} : a_1(\theta_0) = 0, a_2(\theta_0) = 0$ and let $d = [d_1^\top \; d_2^\top]^\top$.

In order to construct the tests we will make the following assumptions, as in Newey and West (1987).

Assumptions: *(i) The data $\{z_t\}_{t=1}^T$ are random vectors that are the first T elements of a strictly stationary stochastic process $\{z_t\}_{t=1}^\infty$ and has a measurable joint density function $f(z_1, ..., z_T, \theta)$ with respect to a measure $\Pi_{t=1}^T \nu$, where ν is a σ-finite measure on $\mathbb{R}^k$.*
(ii) For each $\theta \in \Theta \subset \mathbb{R}^p$, the elements of $g(z, \theta)$ are measurable in z and $\int g(z, \theta) f(z, \theta) d\nu = 0$.
(iii) The vector $g(z, \theta)$ is continuously differentiable on Θ, almost everywhere ν, and $a(\theta)$ is continuously differentiable on Θ. For each positive integer $n \geq 2$ the joint density $f(z_1, z_n, \theta)$ is continuous in θ almost everywhere

$\nu \times \nu$. Also $\theta_0 \in int(\Theta)$ where Θ is compact.

(iv) There exist measurable functions $h_1(z)$ and $h_2(z)$, and $c > 1$, such that almost everywhere ν, and for all $\theta \in \Theta$ and $n \geq 2$,

$$|g(z,\theta)|^4 \leq h_1(z), \quad |\partial g(z,\theta)/\partial\theta|^2 \leq h_1(z),$$

$$f(z,\theta) \leq h_2(z), \quad f(z_1, z_n, \theta) \leq h_2(z_1)h_2(z_n),$$

$$\int [\gamma_1(z)]^c h_2(z) d\nu < +\infty, \quad \int h_2(z) d\nu < +\infty.$$

(v) There exist constants C, $\epsilon > 0$ such that either, (a) for all $\theta \in \Theta$, $\{z_t\}_{t=1}^{\infty}$ is uniform mixing with $\phi(n) \leq Cn^{-\epsilon}$, $\epsilon \geq max\{2, c/(c-1)\}$, (b) for all $\theta \in \Theta$, $\{z_t\}_{t=1}^{\infty}$ is strong mixing with $\alpha(n) \leq Cn^{-\epsilon}$, $\epsilon \geq max\{2, c/(c-1)\}$.

(vi) For all $\theta \in \Theta$, $E[g(z,\theta)] = 0$ only if $\theta = \theta_0$. Also G has rank p, the asymptotic covariance matrix of $\sqrt{T} g_T(\theta_0)$ is nonsingular, and A has rank r.

Let the unconstrained GMM estimator be

$$\hat{\theta}_T = \underset{\theta\in\Theta}{\text{argmax}}\; Q_T(\theta).$$

The assumptions and the results in Newey and West (1987) guarantees that $\hat{\theta}_T$ is consistent and asymptotically normal.

A joint test for H_0^{12} can be constructed based the unconstrained estimator, using a Wald test as a simple application of the delta method. Following Newey and McFadden (1994, p.2220) and the application to time-series data in Newey and West (1987), under H_A^1 and H_A^2, $\sqrt{T}\ a(\hat{\theta}_T) = d + AB^{-1}G\Omega^{-1/2}\mathcal{N} + o_p(1)$ where $\mathcal{N} \sim \mathcal{N}(0_m, I_m)$, so $\sqrt{T}\ a(\hat{\theta}_T) \xrightarrow{d} \mathcal{N}(d, AB^{-1}A^\top)$ as $T \to \infty$. In a similar way, a marginal test for H_0^1 can be constructed without reference to a_2 from $\sqrt{T}\ a_1(\hat{\theta}_T) \xrightarrow{d} \mathcal{N}(d_1, V_{a_1})$ as $T \to \infty$ where V_{a_1} is the asymptotic variance-covariance of $\sqrt{T}a_1(\hat{\theta}_T)$.

Define the constrained GMM estimator as

$$(\bar{\theta}_T, \bar{\lambda}_T) = \underset{\theta \in \Theta, \lambda \in \mathbb{R}^r}{\operatorname{argmax}}\ Q_T(\theta) - a(\theta)^\top \lambda,$$

where λ is the vector of Lagrange multipliers.

The first-order condition (FOC) is

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} q_T(\bar{\theta}_T) - A(\bar{\theta}_T)^\top \bar{\lambda}_T \\ a(\bar{\theta}_T) \end{bmatrix}.$$

Consider the results in Newey and MacFadden (1994, p.2220) under H_A^1 and H_A^2,

$$\begin{bmatrix} \sqrt{T}(\bar{\theta}_T - \theta_0) \\ \sqrt{T}\bar{\lambda}_T \end{bmatrix} = \begin{bmatrix} -B^{-1}A^\top(AB^{-1}A^\top)^{-1} \\ (AB^{-1}A^\top)^{-1} \end{bmatrix} d + \begin{bmatrix} B^{-1/2}MB^{-1/2} \\ (AB^{-1}A^\top)^{-1}AB^{-1} \end{bmatrix} G^\top\Omega^{-1/2}\mathcal{N} + o_p(1),$$

where $M = I - B^{-1/2}A^\top(AB^{-1/2}A^\top)^{-1}AB^{-1/2}$ is a $p \times p$ matrix of rank $p - r$, and $\mathcal{N} \sim \mathcal{N}(0_m, I_m)$.

Define $\Gamma = (AB^{-1}A^\top)^{-1} = [\Gamma_1^\top\ \Gamma_2^\top]^\top$, an $r \times r$ matrix, and

$$\Pi = (AB^{-1}A^\top)^{-1}AB^{-1}G^\top\Omega^{-1/2} = [\Pi_1^\top\ \Pi_2^\top]^\top,$$

an $r \times m$ matrix. Partition of Γ_1 and Γ_2 into the components that correspond to d_1 and d_2, such that:

$$\begin{bmatrix} \sqrt{T}\bar{\lambda}_{1T} \\ \sqrt{T}\bar{\lambda}_{2T} \end{bmatrix} = \begin{bmatrix} \Gamma_{11}d_1 + \Gamma_{12}d_2 \\ \Gamma_{21}d_1 + \Gamma_{22}d_2 \end{bmatrix} + \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} \mathcal{N} + o_p(1).$$

This shows that unless $\Gamma_{12} = 0$, a marginal LM tests for H_0^1 will be contaminated by the validity of H_0^2. This result can be seen as an extension of the results in Davidson and MacKinnon (1987) and Saikkonen (1989) to a GMM framework with general non-linear restrictions.

3 Robust LM test with non-linear hypothesis

Note that d_2 can be written as

$$\bar{d}_{2T} = (I_{r_2} - \Gamma_{22})^{-1}(\sqrt{T}\,\bar{\lambda}_{2T} - \Gamma_{21}d_1 - \Pi_2\mathcal{N}) + o_p(1).$$

Then a robust tests for H_0^1 that is valid for either H_0^2 or H_A^2, can be constructed from

$$\sqrt{T}\,\bar{\lambda}_{1T} - \Gamma_{12}\bar{d}_{2T} = \Gamma_{11}d_1 + \Pi_1\,\mathcal{N} + o_p(1),$$

such that

$$\begin{aligned} \sqrt{T}\left(\bar{\lambda}_{1T} - \Gamma_{12}(I - \Gamma_{22})^{-1}\bar{\lambda}_{2T}\right) &= (\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})d_1 \\ &\quad + (\Pi_1 + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Pi_2)\mathcal{N} + o_p(1). \end{aligned}$$

In particular, a LM test for H_0^1 that is robust to local misspecification in H_0^2 can be constructed as

$$S_{a_1(a_2)}(\lambda) = \sqrt{T}(\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})^{-1}\left(\lambda_1 - \Gamma_{12}(I - \Gamma_{22})^{-1}\lambda_2\right),$$

with

$$S^{\lambda}_{a_1(a_2)}(\bar{\lambda}_T) \xrightarrow{d} \mathcal{N}(d_1, V),$$

where $V = (\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})^{-1}(\Pi_1 + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Pi_2)(\Pi_1 + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Pi_2)^{\top}(\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})^{-1}$.

Also note that the FOC's of the constrained problem, $q_T(\bar{\theta}_T) - A(\bar{\theta}_T)^{\top}\bar{\lambda}_T = 0$, imply that $\bar{\lambda}_T = (A(\bar{\theta}_T)^{\top})^{-}q_T(\bar{\theta}_T)$ where '$-$' denotes the generalized inverse of a matrix. Then $S^{\theta}_{a_1(a_2)}(\bar{\theta}_T) \equiv S^{\lambda}_{a_1(a_2)}(\bar{\lambda}_T)$ is based on the restricted estimator without using the Lagrange multipliers. This transformation has the advantage that in many cases the restricted model is computationally simpler than solving the constrained maximization.

Finally, the LM statistic for H_0^1 that is robust to local misspecification in H_0^2 will be:

$$LM_{a_1(a_2)}(\theta) \equiv S^{\theta}_{a_1(a_2)}(\theta)^{\top}V^{-1}S^{\theta}_{a_1(a_2)}(\theta). \tag{2}$$

The following theorem summarizes the above results.

Theorem 1. *Under Assumption 1 and under H_A^1 and H_A^2*

$$LM_{a_1(a_2)}(\bar{\theta}_T) \xrightarrow{d} \chi^2_{r_1}(d_1^{\top}Vd_1)$$

as $T \to \infty$ where $d_1^{\top}Vd_1$ is the noncentrality parameter and $V = (\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})^{-1}(\Pi_1 + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Pi_2)(\Pi_1 + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Pi_2)^{\top}(\Gamma_{11} + \Gamma_{12}(I_{r_2} - \Gamma_{22})^{-1}\Gamma_{21})^{-1}$.

Remark 1. *Note that in the case that $a_2(\theta_0) = 0_{r_2}$ can be safely assumed,*

$$LM_{a_1}(\bar{\theta}_T) \stackrel{d}{\to} \chi^2_{r_1}(d_1^\top V_1 d_1),$$

where $V_1 = \Gamma_{11}^{-1}\Pi_1\Pi_1^\top\Gamma_{11}^{-1}$.

Remark 2. *In the case where $a(\theta)$ is composed of hypothesis tests of single parameters, i.e. $a_1(\theta) = \theta_1$ and $a_2(\theta) = \theta_2$, then $LM_{a_1(a_2)}(\bar{\theta}_T)$ is equivalent to the LM robust test for specification testing in Bera, Montes-Rojas and Sosa-Escudero (2010). In order to verify this note that A_1 and A_2 become vectors with 1s in the corresponding parameter components and 0 otherwise. Moreover, $\bar{\lambda}_{1T} = q_{1T}(\bar{\theta}_T)$ and $\bar{\lambda}_{2T} = q_{2T}(\bar{\theta}_T)$. Then, the tests correspond to a pseudo-LM test. In fact, if the model is based on a likelihood framework and q are score functions, then the pseudo-scores become scores and the tests are in fact Bera and Yoon (1993) specification tests.*

4 Empirical application: Testing the rational expectations hypothesis

The Mishkin approach

One of the most widely discussed theories in macroeconomics has been the macro rational expectations (RE) hypothesis. This hypothesis, which is due mainly to Lucas (1973), implies that anticipated changes in aggregate demand (e.g., monetary policy) will be incorporated into the economic agents' behavior and will have no effect on real economic activity. Mishkin (1982a,1982b) interprets that the RE implies that the anticipations of aggregate demand will be formed optimally, using all available information. This view became the focus of much empirical investigation.

Since the RE proposition is very controversial, it requires a thorough empirical investigation using a wide range of econometric techniques. Mishkin (1982a,1982b) proposes a feasible statistical and economic model to test for RE.

The basic model consists of a forecasting equation (we use the notation in those papers)

$$X_t = Z_t\gamma + u_t, \tag{3}$$

and an output equation

$$y_t = y_t^* + \sum_{j=0}^{N_1} \beta_j (X_{t-j} - Z_{t-j}\gamma^*) + \sum_{j=0}^{N_2} \delta_j Z_{t-j}\gamma^* + e_t, \tag{4}$$

where X_t is an aggregate demand policy variable (e.g., monetary aggregate), Z_t a vector of macro variables used to forecast X_t that are available at time $t-1$, y_t a real output variable, y_t^* natural level of real output, (u_t, e_t) error terms that are assumed to be uncorrelated with any element of the information set available at time $t-1$, and $\{\beta_j\}_{j=0}^{N_1}$, $\{\delta_j\}_{j=0}^{N_2}$, γ and γ^* are unknown parameters.

Mishkin's model contains in fact two macro assumptions rather than one, that is, RE: $\gamma = \gamma^*$ and neutrality (NE), $\delta_j = 0, \forall j$, which could be stated as the null effect of anticipated changes in aggregate demand policy because economic agents have already take this into account. Mishkin and later Bohara (1991) produce tests for the joint hypothesis of RE and NE, and then marginal tests for RE assuming NE is not necessarily satisfied, but tests for NE assuming RE is satisfied (Mishkin, 1982a, p.26; Mishkin, 1982a, p.791). Mishkin and Bohara both find that the RE hypothesis cannot be rejected, but there is strong support against the NE hypothesis.

In particular, Bohara (1991), following Mishkin (1982a,1982b), proposes a model for which $X_t = m_t$, the log of the first difference of M2, $Z_t = (m_{t-1}, r_{t-1}, r_{t-2}, r_{t-3})$ where r_t is the log of the first difference of the Treasury bill rate. Then, the forecasting equation becomes (Bohara, 1991, p.338, eq. (2))

$$m_t = \gamma_0 + \gamma_1 m_{t-1} + \gamma_2 r_{t-1} + \gamma_3 r_{t-2} + \gamma_4 r_{t-3} + u_t. \quad (5)$$

Consider then the case of $N_1 = N_2 = 1$ such that (Bohara, 1991, p.338, eq. (3))

$$y_t = y_t^* + \beta_0 m_t + \beta_1 m_{t-1} - \beta_0 Z_t \gamma^* - \beta_1 Z_{t-1} \gamma^* + \delta_0 Z_t \gamma^* + \delta_1 Z_{t-1} \gamma^* + e_t. \quad (6)$$

Furthermore assume that $y_t^* = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2}$. In turn this produces a two-equation structural VAR (Bohara, 1991, p.338, eq. (5)),

$$
\begin{aligned}
\begin{bmatrix} m_t \\ y_t \end{bmatrix} &= \begin{bmatrix} \gamma_0 \\ \phi_0 \end{bmatrix} + \begin{bmatrix} \gamma_1 & \gamma_2 & 0 \\ \gamma_1\beta_0 + (\beta_1 - \beta_0\gamma_1^* + \delta_1\gamma_1^*) & \gamma_2\beta_0 + (-\beta_0\gamma_2^* + \delta_0\gamma_2^*) & \phi_1 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ r_{t-1} \\ y_{t-1} \end{bmatrix} + \\
& \begin{bmatrix} 0 & \gamma_3 & 0 \\ (-\beta_1\gamma_1^* + \delta_1\gamma_1^*) & \gamma_3\beta_0 + (-\beta_0\gamma_3^* - \beta_1\gamma_2^* + \delta_0\gamma_3^* + \delta_1\gamma_2^*) & \phi_2 \end{bmatrix} \begin{bmatrix} m_{t-2} \\ r_{t-2} \\ y_{t-2} \end{bmatrix} + \\
& \begin{bmatrix} 0 & \gamma_4 & 0 \\ 0 & \gamma_4\beta_0 + (-\beta_0\gamma_4^* - \beta_1\gamma_3^* + \delta_0\gamma_4^* + \delta_1\gamma_3^*) & 0 \end{bmatrix} \begin{bmatrix} m_{t-3} \\ r_{t-3} \\ y_{t-3} \end{bmatrix} + \\
& \begin{bmatrix} 0 & 0 & 0 \\ 0 & (-\beta_1\gamma_4^* + \delta_1\gamma_4^*) & 0 \end{bmatrix} \begin{bmatrix} m_{t-4} \\ r_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} u_t \\ e_t \end{bmatrix}, \qquad (7)
\end{aligned}
$$

where (u_t, e_t) are white noise.

The null hypothesis of RE involves $H_0^{RE} : \gamma_1 = \gamma_1^*, \gamma_2 = \gamma_2^*, \gamma_3 = \gamma_3^*, \gamma_4 = \gamma_4^*$, and that of NE, $H_0^{NE} : \delta_0 = 0, \delta_1 = 0$. Bohara (1991) considers the nonlinear restrictions that would appear in the restricted VAR under $H_0^{RE} \& H_0^{NE}$, given by

$$
\begin{aligned}
\begin{bmatrix} m_t \\ y_t \end{bmatrix} &= \begin{bmatrix} \gamma_0 \\ \phi_0 \end{bmatrix} + \begin{bmatrix} \gamma_1 & \gamma_2 & 0 \\ \beta_1 & 0 & \phi_1 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ r_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & \gamma_3 & 0 \\ -\beta_1\gamma_1 & -\beta_1\gamma_2 & \phi_2 \end{bmatrix} \begin{bmatrix} m_{t-2} \\ r_{t-2} \\ y_{t-2} \end{bmatrix} + \\
& \begin{bmatrix} 0 & \gamma_4 & 0 \\ 0 & -\beta_1\gamma_3 & 0 \end{bmatrix} \begin{bmatrix} m_{t-3} \\ r_{t-3} \\ y_{t-3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\beta_1\gamma_4 & 0 \end{bmatrix} \begin{bmatrix} m_{t-4} \\ r_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} u_t \\ e_t \end{bmatrix}. \qquad (8)
\end{aligned}
$$

Then he proposes an unrestricted VAR of the form

$$
\begin{bmatrix} m_t \\ y_t \end{bmatrix} = \begin{bmatrix} \theta_{01} \\ \theta_{02} \end{bmatrix} + \begin{bmatrix} \theta_{1,11} & \theta_{1,12} & 0 \\ \theta_{1,21} & \theta_{1,22} & \theta_{1,23} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ r_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & \theta_{2,12} & 0 \\ \theta_{2,21} & \theta_{2,22} & \theta_{2,23} \end{bmatrix} \begin{bmatrix} m_{t-2} \\ r_{t-2} \\ y_{t-2} \end{bmatrix} +
$$

$$
\begin{bmatrix} 0 & \theta_{3,12} & 0 \\ 0 & \theta_{3,22} & 0 \end{bmatrix} \begin{bmatrix} m_{t-3} \\ r_{t-3} \\ y_{t-3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \theta_{4,22} & 0 \end{bmatrix} \begin{bmatrix} m_{t-4} \\ r_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} u_t \\ e_t \end{bmatrix}, \qquad (9)
$$

and tests for the following nonlinear restrictions[2] being zero:

$$
\text{RENE1}: \quad \theta_{2,21} + \theta_{1,11}\theta_{1,21} =
$$

$$
-\beta_1\gamma_1^* + \delta_1\gamma_1^* + \gamma_1\{\gamma_1\beta_0 + (\beta_1 - \beta_0\gamma_1^* + \delta_1\gamma_1^*)\} =
$$

$$
\underbrace{(\beta_1 + \gamma_1\beta_0)(\gamma_1 - \gamma_1^*)}_{RE} + \underbrace{\gamma_1^*(1+\gamma_1)\delta_1}_{NE} = 0,
$$

$$
\text{RENE2}: \quad \theta_{2,22} + \theta_{1,12}\theta_{1,21} =
$$

$$
\gamma_3\beta_0 + (-\beta_0\gamma_3^* - \beta_1\gamma_2^* + \delta_0\gamma_3^* + \delta_1\gamma_2^*) + \gamma_2\{\gamma_1\beta_0 + (\beta_1 - \beta_0\gamma_1^* + \delta_1\gamma_1^*)\} =
$$

$$
\underbrace{\gamma_2\beta_0(\gamma_1 - \gamma_1^*) + \beta_1(\gamma_2 - \gamma_2^*) + \beta_0(\gamma_3 - \gamma_3^*)}_{RE} + \underbrace{\gamma_3^*\delta_0 + (\gamma_2^* + \gamma_2\gamma_1^*)\delta_1}_{NE} = 0,
$$

$$
\text{RENE3}: \quad \theta_{3,22} + \theta_{2,12}\theta_{1,21} =
$$

$$
\gamma_4\beta_0 + (-\beta_0\gamma_4^* - \beta_1\gamma_3^* + \delta_0\gamma_4^* + \delta_1\gamma_3^*) + \gamma_3\{\gamma_1\beta_0 + (\beta_1 - \beta_0\gamma_1^* + \delta_1\gamma_1^*)\} =
$$

$$
\underbrace{\gamma_3\beta_0(\gamma_1 - \gamma_1^*) + \beta_1(\gamma_3 - \gamma_3^*) + \beta_0(\gamma_4 - \gamma_4^*)}_{RE} + \underbrace{(\gamma_3^* + \gamma_1^*\gamma_3)\delta_1}_{NE} = 0,
$$

[2] Note that there are 16 parameters in eq. (7) but only 14 in eq. (9). As such it would not be possible to estimate the parameters first and then to construct Wald-type tests unless additional restrictions are made.

$$\text{RENE4}: \quad \theta_{4,22} + \theta_{3,12}\theta_{1,21} =$$

$$(-\beta_1\gamma_4^* + \delta_1\gamma_4^*) + \gamma_4\{\gamma_1\beta_0 + (\beta_1 - \beta_0\gamma_1^* + \delta_1\gamma_1^*)\} =$$

$$\underbrace{\gamma_4\beta_0(\gamma_1 - \gamma_1^*) + \beta_1(\gamma_4 - \gamma_4^*)}_{RE} + \underbrace{(\gamma_4^* + \gamma_1^*\gamma_4)\delta_1}_{NE} = 0.$$

To these we could add another simple one

$$\text{RENE5}: \theta_{1,22} = \underbrace{\beta_0(\gamma_2 - \gamma_2^*)}_{\text{RE}} + \underbrace{\gamma_2^*\delta_0}_{\text{NE}} = 0.$$

Robust tests

The main issue here is how to construct tests for RE and NE, separately, and that are robust to the presence of the other. Note that not all parameters in each set of hypotheses can be separated from each other. In fact, all of the above equations involve both RE and NE. Note, for instance, that in all cases we have both δ_1 and $(\gamma_1 - \gamma_1^*)$, and as such, deviations from one cannot be separated form the other. The same occurs with δ_0 and $(\gamma_2 - \gamma_2^*)$. As described below, the proposed methodology allows for a test for RE that is robust to the (local) validity of NE.

Our proposed strategy is to estimate the restricted VAR model (8) under the joint null $H_0^{RE\&NE}$, and then to make tests for H_0^{RE} under local misspecification in H_0^{NE}. In particular, we can make the following tests for RE:

1. $H_0^{RE1} : \gamma_3 = \gamma_3^*$. This could be done by considering RENE2 (involving $\gamma_1 = \gamma_1^*, \gamma_2 = \gamma_2^*, \gamma_3 = \gamma_3^*, \delta_0 = 0, \delta_1 = 0$) and local misspecification in RENE1 (involving $\gamma_1 = \gamma_1^*, \delta_1 = 0$) and RENE5 (involving $\gamma_2 = \gamma_2^*, \delta_0 = 0$).

2. $H_0^{RE2} : \gamma_4 = \gamma_4^*$. This could be done by considering RENE4 (involving $\gamma_1 = \gamma_1^*, \gamma_4 = \gamma_4^*, \delta_1 = 0$) and local misspecification in RENE1 (involving $\gamma_1 = \gamma_1^*, \delta_1 = 0$).

3. $H_0^{RE3} : \gamma_3 = \gamma_3^* \& \gamma_4 = \gamma_4^*$. This could be done by considering RENE3 (involving $\gamma_1 = \gamma_1^*, \gamma_3 = \gamma_3^*, \gamma_4 = \gamma_4^*, \delta_1 = 0$) and local misspecification in RENE1 (involving $\gamma_1 = \gamma_1^*, \delta_1 = 0$).

Note that these does not require to assume NE, but rather they allow for local misspecification in NE. This is an alternative test for RE that illustrates the proposed robust testing strategy.

The tests are constructed as follows.

The GMM model can be constructed from the exogeneity assumptions from VAR model (9). In this case

$$\theta = [\theta_{01}\ \theta_{1,11}\ \theta_{1,12}\ \theta_{2,12}\ \theta_{3,12}\ \theta_{02}\ \theta_{1,21}\ \theta_{1,22}\ \theta_{1,23}\ \theta_{2,21}\ \theta_{2,22}\ \theta_{2,23}\ \theta_{3,22}\ \theta_{4,22}]^\top.$$

Define $u_t(\theta) = m_t - (\theta_{01} + \theta_{1,11} m_{t-1} + \theta_{1,12} r_{t-1} + \theta_{2,12} r_{t-2} + \theta_{3,12} r_{t-3})$ and $e_t(\theta) = y_t - (\theta_{02} + \theta_{1,21} m_{t-1} + \theta_{1,22} r_{t-1} + \theta_{1,23} y_{t-1} + \theta_{2,21} m_{t-2} + \theta_{2,22} r_{t-2} + \theta_{2,23} y_{t-2} + \theta_{3,22} r_{t-3} + \theta_{4,22} r_{t-4})$. Then define the $g_t(\theta)$ estimating functions as

$$g_{01,t}(\theta) = u_t(\theta),\ g_{1,11,t}(\theta) = m_{t-1}u_t(\theta),\ g_{1,12,t}(\theta) = r_{t-1}u_t(\theta),$$
$$g_{2,12,t}(\theta) = r_{t-2}u_t(\theta),\ g_{3,12,t}(\theta) = r_{t-3}u_t(\theta),$$
$$g_{02,t}(\theta) = e_t(\theta),\ g_{1,21,t}(\theta) = m_{t-1}e_t(\theta),\ g_{1,22,t}(\theta) = r_{t-1}e_t(\theta),$$
$$g_{1,23,t}(\theta) = y_{t-1}e_t(\theta),\ g_{2,21,t}(\theta) = m_{t-2}e_t(\theta),\ g_{2,22,t}(\theta) = r_{t-2}e_t(\theta),$$
$$g_{2,23,t}(\theta) = y_{t-2}e_t(\theta),\ g_{3,22,t}(\theta) = r_{t-3}e_t(\theta),\ g_{4,22,t}(\theta) = r_{t-4}e_t(\theta).$$

For this model $m = p = 14$. Let $\Omega_T(\theta) = 1/T \sum_{t=1} g_t(\theta)g_t(\theta)^\top$ and construct $g_T(\theta)$ and $G_T(\theta)$ as above.

Note that under $H_0^{RE\&NE}$, model (8) can be easily estimated by first running an OLS model using eq. (5) to get $(\bar{\gamma}_{0T}, \bar{\gamma}_{1T}, \bar{\gamma}_{2T}, \bar{\gamma}_{3T}, \bar{\gamma}_{4T})$, and then to run a second OLS model,

$$\begin{aligned} y_t &= \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} \\ &\quad + \beta_0(m_t - \bar{\gamma}_{1T}m_{t-1} - \bar{\gamma}_{2T}r_{t-1} - \bar{\gamma}_{3T}r_{t-2} - \bar{\gamma}_{4T}r_{t-3}) \\ &\quad + \beta_1(m_{t-1} - \bar{\gamma}_{1T}m_{t-2} - \bar{\gamma}_{2T}r_{t-2} - \bar{\gamma}_{3T}r_{t-3} - \bar{\gamma}_{4T}r_{t-4}) + e_t, \end{aligned}$$

to obtain $(\bar{\phi}_{0T}, \bar{\phi}_{1T}, \bar{\phi}_{2T}, \bar{\beta}_{0T}, \bar{\beta}_{1T})$. These are then used to construct $\bar{\theta}_T$, the set of parameter estimates in the unrestricted VAR model (9).

The tests can then be constructed based on defining the nonlinear restrictions $a_1(\theta)$ and $a_2(\theta)$:

- For H_0^{RE1} define $a_1(\theta) = \theta_{2,22} + \theta_{1,12}\theta_{1,21}$ and $a_2(\theta) = \begin{bmatrix} \theta_{2,21} + \theta_{1,11}\theta_{1,21} \\ \theta_{1,22} \end{bmatrix}$.

- For H_0^{RE2} define $a_1(\theta) = \theta_{4,22} + \theta_{3,12}\theta_{1,21}$ and $a_2(\theta) = \theta_{2,21} + \theta_{1,11}\theta_{1,21}$.

- For H_0^{RE3} define $a_1(\theta) = \theta_{3,22} + \theta_{2,12}\theta_{1,21}$ and $a_2(\theta) = \theta_{2,21} + \theta_{1,11}\theta_{1,21}$.

From these $LM_{a_1(a_2)}(\bar{\theta}_T)$ can be constructed in each case.

Data, constrained parameter estimates and test results

Table 1 presents the regression estimates of the forecasting and output equations. The data corresponds to that outlined by Bohara (1991), with the series extended to 2015.

We consider first the individual LM tests for the hypotheses RENE1-RENE5 in Table 2. The tests are constructed for the 1959-1986 and 1959-2015 subsamples. In both subsamples the tests show that the RENE1 is rejected, but we cannot reject any of the remaining RENE2-RENE5 hypotheses. This determines that either $\gamma_1 \neq \gamma_1^*$ or $\delta_1 \neq 0$.

Table 3 reports the LM robust tests for RE1-RE3. In this case, the results point that $\gamma_3^* = \gamma_3$ in RE1 cannot be rejected, but there is some evidence that $\gamma_4^* \neq \gamma_4$ in RE2 for the 1959-1986 subsample and marginal evidence for 1959-2015. RE3 offers clear rejections of the null hypothesis in both cases. Exploring these results might reveal that it is likely that both γ parameters do not satisfy the RE restriction. This is due to the statistical significance of the parameter estimates in the reduced form estimates under the joint null. In particular, the results indicate that β_0 is not statistically different from zero, while β_1 is. RE1 is based on RENE2, where $(\gamma_3 - \gamma_3^*)$ is multiplied by β_0, which is not significant in any subsample in Table 1. However, RE2 (based on RENE4) has β_1 multiplied by $(\gamma_4 - \gamma_4^*)$, while RE3 (based on RENE3) has β_1 multiplied by $(\gamma_3 - \gamma_3^*)$.

In summary, our developed methodology reveals that we are able to reject the RE hypothesis, while the previous work by Bohara and Mishkin agreed on its validity.

5 Conclusion

This paper extends the Bera and Yoon (1993) testing LM robust strategy to time-series dependent data and general linear and nonlinear functions for the hypothesis of interest and locally misspecified alternatives. The tests are asymptotically chi-squared with degrees of freedom given by the dimensions of the null hypothesis of interest, and with noncentrality parameter that correspond to the null of interest and not to the misspecified alternative.

The possibility of using linear and nonlinear functions multiplies the number of potential applications. As an example......

Extensions....

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Table 1: Parameter estimates

	Forecasting equation			Output equation	
	1959-1986	1959-2015		1959-1986	1959-2015
$\bar{\gamma}_{0T}$	.0125***	.0071***	$\bar{\phi}_{0T}$	.0053***	.0040***
	(.0018)	(.0010)		(.0014)	(.0008)
$\bar{\gamma}_{1T}$	.4011***	.5741***	$\bar{\phi}_{1T}$	.2554***	.2635***
	(.0084)	(.0560)		(.0942)	(.0655)
$\bar{\gamma}_{2T}$	-.0201***	-.0029	$\bar{\phi}_{2T}$	.1247	.1987***
	(.0046)	(.0020)		(.0942)	(.0655)
$\bar{\gamma}_{3T}$	-.0050	.0035*	$\bar{\beta}_{0T}$	-.0092	.0256
	(.0048)	(.0020)		(.1451)	(.0756)
$\bar{\gamma}_{4T}$	-.0096**	-.0010	$\bar{\beta}_{1T}$	.3793***	.2052***
	(.0047)	(.0020)		(.1451)	(-0754)

Notes: OLS estimates. Standard OLS standard errors in parentheses. * Significant at 10%. ** Significant at 5%. * Significant at 10%.

Table 2: LM standard tests

	1959-1986	1959-2015
	RENE1	
LM stat	281.6962	8.0091
p-value	[.0000]	[.0047]
	RENE2	
LM stat	.0890	.1496
p-value	[.7654]	[.6989]
	RENE3	
LM stat	.6276	.0917
p-value	[.4282]	[.7621]
	RENE4	
LM stat	1.5624	.2458
p-value	[.2113]	[.6200]
	RENE5	
LM stat	2.2464	.0633
p-value	[.1339]	[.8013]

Notes: All statistics are central chi-squared with 1 degree of freedom under the null hypothesis.

Table 3: LM robust tests

	1959-1986	1959-2015
	RE1: $\gamma_3 = \gamma_3^*$	
LM stat	2.0907	.1276
p-value	[.1482]	[.7209]
	RE2: $\gamma_4 = \gamma_4^*$	
LM stat	6.0695	2.6152
p-value	[.0138]	[.1058]
	RE3: $\gamma_3 = \gamma_3^* \& \gamma_4 = \gamma_4^*$	
LM stat	3.7667	6.3475
p-value	[.0523]	[.0118]

Notes: All statistics are central chi-squared with 1 deree of freedom under the null hypothesis.