Monetary Methods to Measure the Size of the Shadow Economy: A Critical Assessment

Hildegart Ahumada (Universidad Torcuato Di Tella)
hahumada@utdt.edu
Facundo Alvaredo (PSE, Paris and CONICET)
alvaredo@pse.ens.fr
Alfredo Canavese (Universidad Torcuato Di Tella and CONICET)
alca@utdt.edu

JEL classification: H3 –K4

Abstract

The monetary method is a widely used approach to measure the size of the shadow economy. It is based on the assumption that cash is used to make transactions that agents want to keep hidden from official records This paper provides a formal aggregation framework which stylizes the steps followed in empirical applications and demonstrates that it only produces coherent results if the income-elasticity of currency demand is one. The paper also suggests a way to correct the estimated size of the shadow economy when that requirement is not fulfilled. Besides, the paper shows why short-run estimates should be based on previous values of the size of the shadow economy.

Monetary Methods to Measure the Size of the Shadow Economy: A Critical Assessment[1]

Hildegart Ahumada (Universidad Torcuato Di Tella)
Facundo Alvaredo (PSE, Paris and CONICET)
Alfredo Canavese (Universidad Torcuato Di Tella and CONICET)

"…cash leaves no tracks, and makes no demands on anybody else's integrity."
Benjamin Friedman, The Economist, July 22, 2000, p.76[2]

National accounts do not register a whole set of economic transactions. Undeclared, under-declared, non-measured and under-registered transactions made to avoid the burden of taxes or to circumvent regulations, illegal transactions connected with crime and corruption and legal but non-market activities are included in the concept of shadow economy. For the last thirty years economists have been interested in the study of these transactions under the names of hidden, unrecorded, underground, parallel, black or shadow economy. A special volume of The Economic Journal (1999) and a survey by Schneider and Enste (2000) thoroughly document such interest.

The average estimated size of the shadow economy varies from 12% of registered GDP for OECD members to 23% for transition economies and to 39% for developing countries.

The very nature of the shadow economy makes its measurement a difficult task. Furthermore, different estimation methods target different concepts[3]. So, estimation methods have become an important theoretical and empirical issue.

A widely used approach to measure the size of the shadow economy - known as "the monetary method"- is based on the assumption that cash is used to make transactions that agents want to keep hidden from official records. Transactions made using cash are difficult to trace: they leave no tracks. If the amount of currency used to make hidden transactions can be estimated, then this amount could be multiplied by the income-velocity of money to get a measure of the size of the shadow economy. The monetary approach was first presented by

[1] We thank E. Schneider and T. Breusch for useful comments. We gratefully acknowledge the assistance of Alejandro Francetich and Juan Pablo Xandri.
[2] Leonardo Gasparini kindly pointed-out this reference to the authors.
[3] Authors do not seem to be always aware of this observation.

Gutman (1977) and Feige (1979) and it has evolved to use econometric tools in estimates made by Tanzi (1982, 1983). Empirical work has been done for many countries. We analyze the monetary method to show that to get coherent estimates of the size of the shadow economy the income elasticity of the demand for money should meet some requirements that most published works do not fulfill.

The paper has six sections. In the first section we review the evolution of the monetary method used to compute the size of the shadow economy. We devote the second section to build a formal aggregation framework which stylizes the steps followed in the empirical applications of the method and to derive the conditions that the income-elasticity of the demand for real money balances should meet to produce coherent estimates. In section three we suggest a way to correct the estimated size of the shadow economy when those conditions are not fulfilled. Then we apply the corrections to existent measures for different countries. We devote the fifth section to analyze conditions that short run estimates should fulfill to obtain the size of the shadow economy. We make some closing remarks in the last section.

1. A review of the monetary approach evolution

The monetary approach to measure the size of the shadow economy is based on the assumption that cash is used to make transactions that agents want to keep hidden from official records. Transactions made using cash are difficult to trace because they leave no tracks. Other assets are registered in financial institutions and their uses are recorded in such a way that transactions made with them can be easily inspected. If the amount of currency used to make hidden transactions can be estimated, then this amount could be multiplied by the income-velocity of money to get a measure of the size of the shadow economy.

The monetary approach was first presented by Gutman (1977) and Feige (1979) and it has evolved to use econometric tools in estimates made by Tanzi (1982, 1983), which are based on Cagan (1958). The technique was then applied to measure the size of the shadow economy in the US, Italy, Norway, Canada, South Africa, Tanzania, Mexico, India, Australia, Austria, Belgium, Denmark, France, Germany, Great Britain, Ireland, Netherlands, New Zealand, Spain, Sweden, Switzerland Argentina, etc.[4]

Gutman's method (1977) is based on four key assumptions: (a) high taxes and government regulations are the main causes of the existence of a shadow sector, (b) only cash is used to make transactions in the shadow economy, (c) the ratio of currency to demand deposits, *C/D*, is only influenced by changes in taxes and regulations, and (d) there was some point in time in the past when no shadow economy existed. As the ratio *C/D* of that period should have prevailed except for changes in the level of taxes and regulations, increases in *C/D* are directly linked to the extra currency used in the shadow economy. The method assumes that the income-velocity of circulation, v, is equal for the registered and the hidden economies; hence the size of the hidden sector is v times the extra currency.

Feige's method (1979) uses the standard version of the quantity theory of money $Mv = PT$, where M is money including demand deposits. The value of transactions is PT. Assuming that the ratio of the value of transactions to nominal income remains constant through time and that it is known for a period in which there were no hidden transactions then, total nominal income can be estimated for any period. The difference between estimated total nominal income and observed nominal income is the size of the shadow economy. Feige assumes that hidden transactions are made using either cash or checks.

Gutman and Feige inaugurated two traditions in the use of monetary methods to measure the size of the shadow economy. Both assume that only some monetary aggregates are used to finance hidden transactions but they differ in the way the money used in the shadow economy is reckoned: Gutman's work is based in a portfolio selection approach while Feige's paper uses a money demand calculation.

The work by Tanzi (1980, 1982, 1983) and all the papers based on his approach perform econometric estimates of the monetary aggregates used to finance hidden transactions. The introduction of econometric tools allows recognizing that the income-velocity depends not only on variables that induce economic agents to make hidden transactions but also on income and the opportunity cost of holding cash. The estimated equation of the demand for

[4] Tanzi (1999) gives a skeptical view given the wide diversity of the results obtained.

currency is also useful to get the extra cash held by economic agents to finance hidden transactions without postulating that there was some point in time in the past when no shadow economy existed. It is again assumed that the income-velocity of circulation for registered and hidden transactions is equal, and so the size of the shadow economy is measured multiplying the extra cash by the econometrically obtained value of v.

2. An Aggregation Framework

We devote this section to build a formal aggregation framework for the monetary method based on the econometric estimation of the demand for currency defined as real cash holdings. It is also employed to derive the condition that the income-elasticity should meet to obtain equal velocities in both sectors.

A currency demand function in Cagan's (1958) tradition can be expressed as:

$$C_O = A\left(1+\Theta\right)^{a}\ Y_O^{b}\ \exp(-g\,i) \qquad (1)$$

where C_O denotes observed cash balances, Θ is a vector of variables which induce agents to make hidden transactions (for example the ratio of taxes or government expenditure to GDP), Y_O is a scale variable (for example registered GDP), i measures the opportunity cost of holding cash (the interest rate or the rate of inflation); A, a, b and g are positive parameters. Observed currency, C_O, is equal to total currency, C_T, which includes cash used for recorded transactions, C_R, plus cash used for hidden transactions, C_H,

$$C_O = C_T = C_R + C_H \qquad (2)$$

Observed GDP, Y_O , is the real registered GDP, Y_R , which does not include hidden GDP, Y_H ,

$$Y_T = Y_O + Y_H = Y_R + Y_H \tag{3}$$

Since observed currency includes C_H but observed GDP excludes Y_H, the usual econometric regression of C_O on Y_O would result in biased and inconsistent estimates.

The empirical applications based on this technique follow four steps. First, a demand for currency is estimated as in (1).[5] Second, under the assumption that the demands for C_R and C_H have the same functional form with equal parameters, Θ is set equal to zero to get an estimate of the amount of cash demanded under no incentives to hide transactions, $\hat{C}_R$,

$$\hat{C}_R = \hat{A} Y_O^{\hat{b}} \exp(-\hat{g}\, i) \tag{4}$$

Third, since $\hat{C}_R$ is known from (4) and C_T is observed currency, C_O , $\hat{C}_H$ can be obtained by difference,[6]

$$\hat{C}_H = C_T - \hat{C}_R \tag{5}$$

Fourth, to get the size of the shadow economy it is assumed that the velocity of circulation for both, registered and hidden transactions, is the same, so

$$v_R = \frac{Y_R}{C_R} = \frac{Y_H}{C_H} \tag{6}$$

and then,

[5] To take into account that the time series are integrated, some works consider equation (1) as a long run relation. Other papers estimate first difference equations, partial adjustment models or hybrids.

[6] Tanzi (1982) uses $\hat{C}_T$ instead of C_T in (5).

$$\hat{Y}_H = \hat{v}_R \hat{C}_H \quad (7)$$

$\hat{Y}_H$ is ‘the’ estimation of the size of the shadow economy and it is obtained using $\hat{C}_H$ from (5) and $\hat{v}_R$ from (7).

So far we have described the procedures followed in the literature. The key assumption made explicit in (6) requires $b = 1$, which is evident if we recall that the income-velocity for registered money is

$$v_R = \frac{Y_R}{C_R} = \frac{Y_R}{AY_R^b \exp(-gi)} = \frac{{Y_R}^{1-b}}{A\exp(-gi)} \quad (8)$$

while the velocity for the hidden economy is

$$v_H = \frac{Y_H}{C_H} = \frac{Y_H}{AY_H^b \exp(-gi)} = \frac{{Y_H}^{1-b}}{A\exp(-gi)} \quad (9)$$

The velocity is the same in both sectors if $b = 1$.[7] Those studies that find $\hat{b} \neq 1$ but follow the steps described above are therefore incorrect.[8]

The assumption that the demands for C_R and C_H follow Cagan’s type forms with equal parameters allow an explicit aggregation of (2) as:

$$C_T = AY_R^b \exp(-gi) + AY_H^b \exp(-gi) = AY_R^b \exp(-gi)\left(1 + \left(\frac{Y_H}{Y_R}\right)^b\right) \quad (10)$$

---

[7] The velocity is also the same for the improbable case in which $Y_R = Y_H$ for any b.

[8] It should be stressed that currency is the money aggregate whose demand should have income elasticity equal to one. While this value may appear reasonable and theory-based in the case of the demand for the aggregate used to finance all transactions (e.g. the demand for M_1), it may not necessarily be correct for narrower definitions of money. For instance, in Baumol-Tobin’s model, the value of transactions elasticity is 1/2.

This formulation does not need to be restricted to currency. It is also valid for any wider aggregate (e.g. M_1) as long as the interest rate or the inflation rate is the opportunity cost of holding it. However b obtained from (1) only matches b in (10) if the ratio $\frac{Y_H}{Y_R}$ is independent of Y_R.[9] It must be stressed that all papers using this approach implicitly make this assumption.

3. A Suggested Correction

In this section we show that there is no need to impose ad hoc restrictions on the velocity of circulation when the currency demand approach is followed to compute the size of the hidden economy. The problem addressed in section 2 can be solved by an explicit recognition that v depends on the value of b. We also suggest a way to "correct" wrong estimates made by imposing $b = 1$ when it is not the case.

Equation (10) can be written as:

$$C_T = AY_R^b \exp(-gi)(1+\Theta)^a \tag{11}$$

which is equal to (1).

Recalling that $Y_O = Y_R$, $C_T = C_O$ and that C_T and Y_R are observed variables, (11) can be econometrically estimated as in (1).[10] Next, as we already described in the previous section, setting Θ=0 provides an estimate for C_H. The ratio between C_R and C_H is

$$\frac{C_R}{C_H} = \frac{AY_R^b \exp(-gi)}{AY_H^b \exp(-gi)} = \left(\frac{Y_R}{Y_H}\right)^b \tag{12}$$

---

[9] For this reason it is convenient to measure Θ normalized by registered GDP.

[10] The variable $(1+\Theta)^a$ is sensitive to changes in the units in which Θ is measured, as pointed out in Breusch (2005a) and (2005b) . We keep the notation to maintain Tanzi's original functional form. Nevertheless, changing $(1+\Theta)^a$ for $(\exp(a\Theta)-1)$ could solve this problem.

Equation (12) provides an expression for Y_H given Y_R, C_R, C_H and b. Consequently there is no need to make the ad-hoc assumption on the equality of income velocity in both sectors.

The assumption about the equality of v for hidden and registered transactions is accurate if $b = 1$. It can also be seen from (12) since,

$$\frac{Y_R}{C_R} = \left(\frac{Y_R}{Y_H}\right)^{1-b} \frac{Y_H}{C_H} \qquad (13)$$

or

$$v_R = \left(\frac{Y_R}{Y_H}\right)^{b-1} v_H \qquad (14)$$

Equation (14) shows in another way that it is inaccurate to assume that v is equal for registered and hidden transactions when the hypothesis $b = 1$ is rejected. Equation (12) can be used to "correct" wrong estimates made by imposing $b = 1$ when it is not the case. From (12) it follows that

$$\frac{C_R}{C_H} = \frac{\overline{Y}_R}{\overline{Y}_H} \qquad (15)$$

where $\frac{\overline{Y}_R}{\overline{Y}_H}$ is the (faulty) ratio obtained under the restriction $b = 1$. From (12) and (15)

$$\frac{Y_H}{Y_R} = \left(\frac{C_H}{C_R}\right)^{\frac{1}{b}} = \left(\frac{\overline{Y}_H}{\overline{Y}_R}\right)^{\frac{1}{b}} \qquad (16)$$

which shows how to correct results obtained assuming $b = 1$ when $b \neq 1$.

It is worthwhile noting that when the ratio of two monetary aggregates is econometrically estimated instead of a money demand equation the same objection discussed above remains unsolved. Once money used to finance non-registered transactions is calculated, the assumption of equal velocities of circulation for registered and hidden currencies is again made to get the size of the shadow economy. In fact, velocity should be "known" to get transactions. The critical point is that velocity depends on the income elasticity of money demand which is an empirical issue. Indeed, this case is even worse since no estimations of such elasticity are provided by modeling ratios.[11] Sometimes, modeling ratios entails another measurement error when it is not recognized that the absence of shadow transactions may affect the whole ratio and not only one monetary aggregate. In Appendix 1 we analyze this case.

4. Corrected Values for Some Empirical Estimations

In this section we present some results only as an exercise to illustrate the main point of the paper. We do not intend to provide accurate measures of the size of the underground economy. We only show estimates from other papers to apply the correction described above.[12] In fact it should be pointed out that the level of econometric analysis in these studies is rather basic and casts some doubts on its results especially when the key role played by the estimated income elasticity is duly recognized[13]. Authors generally present their results as percentages of GDP, implying that they computed the portion of GDP not registered by statistics.

[11] It should be noticed that a significant effect of transactions (as in Tanzi´s econometric equation) indicates different elasticities for the aggregates entering the ratio and thus different velocities.

[12] Unfortunately, many studies applying the monetary method do not show the results of the estimation of the demand for currency, and they focus on the "one" number: the size of the shadow economy as a percentage of the GDP.

[13] A pair of examples illustrates the point. Thomas (1989) re-estimated Tanzi's model for 1930-1980 and found evidence of a structural break in 1945 while the tax variable was statistically not significant after 1946. Tanzi (1982) had found an income elasticity very close to one. Smith (1986) showed that the model of Matthews and Rastogi (1985) was mis-specified.

Argentina 1930-1983

Guisarri (1986) measures the size of the shadow economy in Argentina for 1930-1983 using annual data. He estimates a demand for currency which could be understood as a long run equation. The share of government expenditure in GDP and the ratio between the official and black exchange rates of the U.S. dollar were the chosen variables to quantify the incentives to hide transactions in Θ. The econometric estimate for $\hat{b}$ is 0,508. He follows the standard technique described in section 3, that is, the assumption of equal velocities or $b = 1$. According to his calculations, the size of the shadow economy in 1983 (his last observation) represented 56% of registered GDP. Nevertheless, our correction implies that the magnitude of the hidden sector was 32%.

Australia 1967-2000

Bajada and Schneider (2003) produce a time series estimate of the 'cash economy' in Australia between 1967 and 2000. The results are based on an error correction model for money demand in which $\hat{b} = 0,852$. This is the long run estimate obtained by assuming the static equilibrium value of all variables. They calculate that the size of the shadow economy in Australia between 1990 and 2000 averaged 14,6% of GDP. They also compare this figure with the one resulting from a MIMIC approach: 14,82%. They stress the similarity of estimates. However, if the method is correctly followed, the fraction of the shadow income was 10,4% of the registered economy.

Norway 1952-1978

Isachsen et al. (1982) get an income elasticity of the demand for currency of 0,85 between 1952 and 1978. The correction changes the share of the shadow economy from 8% to 5,1% of the registered economy.

Isachsen and Strom (1985) use the currency demand approach to get an estimate of the hidden economy of 6,3% for 1978. This estimate is based on a partial adjustment currency demand equation with a long run income elasticity equal to 0,663. The corrected estimate gives 1,51%.

Austria 1956-1998

Schneider (2000) applies the monetary method to Austria for the period 1956-1998. A partial adjustment currency demand equation is estimated as a function of lagged currency, the number of Eurochecks per capita, the real interest rate on bonds, a measure of the indirect tax burden, an index of regulations and per capita consumption as a proxy for the income effect. Although the reported short-run income elasticity is 0,734, the implicit long-run value is $\hat{b}$ =2,04. The author calculates that the size of the shadow economy was 9,12% of registered GDP in 1998. The correction gives 30,9%.

Tanzania 1968-1990

Bagachwa and Naho (1995) present two different time series estimates of the shadow economy, based on different real currency demand functions which differ only in the way the tax and government intervention variables enter the equation. The ratio of hidden to registered economy for 1990 is 33,24% from the first equation and 20,96% from the second. However, the income elasticities they find are 2,323 and 2,569 respectively. For 1990 the corrections give estimates of 62,2% and 54,4%.

5. Short-run vs. Long run estimates

Econometric models of money demand usually include the lagged dependent variable as regressor. These dynamic models should often be formulated to take into account the time properties of the series involved (real cash holdings, income, interest rates, inflation, etc.) which have resulted either integrated or -at least- highly persistent. From "partial adjustment" to "equilibrium correction", all models including the lagged dependent variable admit two types of estimates: one for the short-run and another for the long-run. When the first kind of estimates are used to obtain the size of the shadow economy an additional problem arises, which will be discussed in this section.

Before that, it is worthwhile noting that in section 2 we derived the solution of the hidden economy estimates as an aggregation problem based on a Cagan-type money demand. As no dynamic structure is allowed for, the estimates can be seen as those belonging to the long run solution. However, when dynamics are introduced by including the lagged dependent variable, an appropriate calculation requires the size of the hidden economy for a past period (e.g. a period without illegal transactions) to be known. No need of such “knowledge” was just a remarkable advantage of the econometric approaches with respect to previous methods. In order to appreciate this issue, we consider a usually estimated log form of regression that includes the lagged dependent variable,

$$c_{Tt} = b_1 + b_2 c_{Tt-1} + b_3 \ln(y_{rt}) + b_4 i_t + b_5 \ln(1+q_t) \tag{17}$$

where c_{Tt} stands for log of total currency demand, y_{rt} stands for the log of registered (observed) output, i_t represents the opportunity cost of holding currency and finally Θ_t is a variable that induces people to hide activities.

Equation (17) can be seen as a model of partial adjustment to reach:

$$C_{Tt} = A y_{rt}^{b} \exp(-g i_t)(1+q_t)^{a}$$

which is the same as equation (12),

or in logs:

$$c_{Tt} = \ln(A) + b \ln(y_{rt}) - g i_t + a \ln(1+q_t) \tag{18}$$

as the equilibrium for the desired level of real cash holdings. Equation (17) has the following long-run solution

$$c_{Tt} = c_{Tt-1}$$

$$c_{Tt} = \frac{b_1}{1-b_2} + \frac{b_3}{1-b_2}\ln(y_{rt}) + \frac{b_4}{1-b_2} i_t + \frac{b_5}{1-b_2}\ln(1+q_t) \tag{19}$$

It can be readily seen from (18) y (19) that:

$$\frac{b_1}{1-b_2}=\ln(A); \quad \frac{b_3}{1-b_2}=b; \quad \frac{b_4}{1-b_2}=-g; \quad \frac{b_5}{1-b_2}=a$$

In this case it is straightforward to show that when $\Theta_t = 0$, c_{rt} can be directly obtained. However, it is a quite different issue to obtain c_{rt} for the short run, for example, from equation (17), since c_{rt-1} or a previous value of it should be required. In order to further analyze this issue it is convenient to show the kind of aggregation assumed in the partial adjustment model, which should be formulated for the level of the composite good ($C_T = C_r + C_h$) in order to get equation (17) taking logs.[14]

$$C_{Tt} = C^{l}_{Tt-1} \cdot C^{*(1-l)}_{Tt} = (C_R + C_H)^{l}_{t-1} \cdot (C_R + C_H)^{*(1-l)}_{t} \tag{20}$$

where

$$C_{Tt}^{*} = Ay_{rt}^{b}\exp(-g\,i_t)\left(1+\left(\frac{y_{ht}}{y_{rt}}\right)^{b}\right) = Ay_{rt}^{b}\exp(-g\,i_t)(1+q_t)^{a} \tag{21}$$

When $\Theta_t = 0$, only the part of C^{*}_{rt} (target or desired) due to registered transactions can be obtained but not C_{rt} as a whole since C_{rt-1} should also be known. When, as often it has been done in applied work, C_{Tt-1} is used instead, C_{rt} is overestimated as l estimates are positive. If C_{rt-1} were substituted by C_{rt-2} and C_{rt-2} by its previously value and so on, the required knowledge about registered currency would be moved to some previous value or eventually to an initial value C_{r0}, as can be seen by solving the corresponding first difference equation for the log of currency demand

---

[14] If the partial adjustment were assumed for each component (C_r and C_h) in equation (17), this equation would not allow us to estimate the share Y_r / Y_h as it cannot be isolated from $Y_r C_{rt-1} / Y_h C_{ht-1}$ unless $C_{rt-1} = C_{ht-1}$.

To sum up, the only way to avoid ad hoc assumptions about previous values of registered currency is to restrict the measures of the shadow economy size to those based on the long run estimates of the money demand.

6. Final Remarks

The monetary method to measure the size of the shadow economy is based on econometric estimates of the demand for money. These estimates are used to get the currency held by economic agents in excess of the amount they need to finance registered transactions. The standard monetary approach uses the excess of currency multiplied by the velocity of circulation (assumed to be equal in the registered and the shadow economies) to measure hidden GDP.

This paper builds a formal aggregation framework to show that this procedure is accurate only when the income elasticity of the demand for money is one. The paper also suggests a way to correct faulty estimates and applies it to some published results to show that the assumption of equal velocities together with income elasticity estimates lower (higher) than one result in figures biased upwards (downwards) for the shadow economy.

Moreover, short run estimates obtained using econometric models require the size of hidden currency from a previous period to be known. As a consequence a main advantage of using econometrics (viz. no need of assuming a basis period without illegal transactions) is lost.

For results based on econometric estimates of the ratios of currency to broad money or currency to deposit a similar problem is found. Accurate measures of the size of the shadow economy require unit income elasticity. However, in this approach no estimation of this elasticity is explicitly provided.

Appendix 1

In this appendix we discuss a particular point that appears when econometric estimates of the ratio of two monetary aggregates are used to compute the size of the shadow economy. Sometimes it is not recognized that the absence of shadow transactions affects the whole ratio and not only one monetary aggregate. It is the case when the ratio of currency to broad money is used as in Tanzi (1980, 1983).

Tanzi follows three steps to estimate the size of the shadow economy.

First, from the estimates of the econometric model, the value of the ratio of currency to money (M_2) in absence of incentives to deviate resources to the hidden sector is calculated. Let this value be $(\frac{C}{M_2})_R$. Tanzi calculates registered currency C_R^* multiplying this ratio by total money holdings:

$$C_R^* = (\frac{C}{M_2})_R M_2 = C_R \frac{M_2}{M_{2R}} \tag{A1}$$

However, in absence of hidden economy, the ratio estimated by the model stands for the ratio of registered currency to registered money, not total money. This introduces a measurement error as hidden currency is part of M_2

$$M_2 = M_{2R} + C_H \tag{A2}$$

then for $C_H > 0$

$$\frac{M_2}{M_{2R}} = 1 + \frac{C_H}{M_{2R}} > 1 \tag{A3}$$

Hence, this method overestimates registered currency when the ratio $(\frac{C}{M_{2R}})$ is multiplied by M_2 instead of M_{2R}

$$C_R^* = C_R(1+\frac{C_H}{M_{2R}}) > C_R \qquad \text{(A4)}$$

Second, hidden currency is calculated from the difference between total currency and registered currency,

$$C_H^* = C_T - C_R^* \qquad \text{(A5)}$$[15]

and

$$C_H^* = C_H(1-\frac{C_R}{M_{2R}}) < C_H \qquad \text{(A6)}$$

Consequently, as registered currency is overestimated, hidden currency is underestimated. So

$$C_H^* < C_H \qquad \text{(A7)}$$

Third, the size of the hidden economy is obtained using legal money (M_1) velocity, which is assumed to be equal to money velocity in the underground economy. However, legal money velocity is underestimated because:

$$M_{1R}^* = M_{1o} - C_H^* > M_{1R} \qquad \text{(A8)}$$

Hence,

$$v^* = \frac{Y_R}{M_{1R}^*} < \frac{Y_R}{M_{1R}} = v \qquad \text{(A9)}$$

Therefore, we conclude that:

[15] Tanzi (1982) uses estimated C_T

$$Y_H^* = v^* C_H^* < vC_H = Y_H \qquad \text{(A10)}$$

The difference between Y_H and Y_H^* depends, in principle, on the difference between C_H and C_H^*. This difference can be calculated through the ratio $\frac{C_R}{M_{2R}}$ as it is shown in equation (*) . The ratio $\frac{C_R}{M_{2R}}$ is precisely the ratio calculated in the first step of the method. Therefore, the estimates of legal money velocity, hidden currency and hidden economy can be corrected.

The next table shows for some years the corrected values of the estimates of the hidden economy for the two cases analyzed in Tanzi (1983): using a weighted average tax rate (WT) and an average tax rate (T) in the currency ratio model respectively.

Table 2. Correction of shadow economy estimates from C/M2

	TW		**T**	
Year	**Yh*/Y**	**Yh/Y**	**Yh*/Y**	**Yh/Y**
1938	1,332	1,496	0,459	0,536
1946	4,480	5,307	2,245	2,703
1960	4,076	4,620	2,633	3,011
1980	6,069	6,681	4,495	4,982

Values are expressed as percentages

Although the size of the correction would be small, the problem can be avoided if followers of the portfolio approach model the ratio of currency to deposits instead of currency to broad money, as done in Mirus, et. al. (1994). Deposits are assumed not to include money holdings for hidden transactions. In both cases the issue about the income elasticity pointed out in the text remains unsolved.

References

Bagachwa, M. and Naho, A., "Estimating the second economy in Tanzania", *World Development,* 23:8

Bajada, C. and Schneider, F. (2003), "The Size and Development of the Shadow Economy in the Asia-Pacific", Economics WP, 2003-1. Department of Economics, Johannes Kepler University Linz, Austria.

Breusch, T. (2005a), "Australia's Cash Economy: Are the estimates credible?", The Australian National University, mimeo.

Breusch, T. (2005b), "Fragility of Tanzi's method of estimating the underground economy", The Australian National University, mimeo.

Cagan, P. (1958) "The Demand for Currency Relative to the Total Money Supply", *Journal of Political Economy*, 66:3.

Feige, E. (1979) "How Big is the Irregular Economy?"; *Challenge*, 22:1.

Guissarri, A. (1986) "La Demanda de Circulante y la Informalidad en la Argentina: 1930 – 1983", Seminarios ITDT.

Gutmann, P. (1977) "The Subterranean Economy", *Financial Analysts Journal*, 34:1.

Isachsen, A., Klovland, J. and Strom, S. (1982) "The Hidden Economy in Norway", in *The Underground Economy in the United States and Abroad*, V. Tanzi (ed.), Lexington Books.

Isachsen, A.and Strom, S. (1985) "The Size of the Hidden Economy in Norway", *Review of Income and Wealth*, vol. 31, issue 1, pages 21-38.

Matthews, K and Rastogi, A. (1985) "Little Mo and the moonlighters: another look at the black economy", Liverpool Research Group in Macroeconomics *Quarterly Economic Policy*, 8:2.

Mirus, R. Smith R. and Karoleff V. (1994), "Canada´s Underground Economy Revisited: Update and Critique", *Canadian Public Policy- Analyse de Politiques*, XX. Pages 235-252

Schneider, F. (2000), "The Increase of the Size of the Shadow Economy of 18 OECD Countries: Some Preliminary Explanations", CESifo WP 306.

Schneider, F. and Enste, D. (2000) "Shadow Economies: Size, Causes, and Consequences", *Journal of Economic Literature*. 38:1.

Tanzi, V. (1982) "Underground Economy and Tax Evasion in the United States: Estimates and Implications", in *The Underground Economy in the United States and Abroad*, V. Tanzi (ed.); Lexington Books.

Tanzi, V. (1983) "The Underground Economy in the United States: Annual Estimates, 1930-80", *IMF Staff Papers*, 30:2.

Tanzi, V. (1999) "Uses and Abuses of Estimates of the Underground Economy"; *Economic Journal*, 109.

Thomas, J. (1986) "The underground economy in the United States: a further comment on Tanzi;" *Staff Papers, IMF*, 33.