Abstract

This paper examines how political budget cycles (PBC) are affected by checks and balances when the political parties are office motivated. When the legislature has to authorize new debt, there are no PBC if there is divided government and perfect compliance with the budget law. PBC are only possible when there is unified government or low compliance with the budgetary law. What drives these results are effective checks and balances, that provide a commitment device to solve the credibility problem behind PBC. An extension of the basic model is to analyze the tradeoff voters face when divided government comes at the cost of government efficiency.

JEL Classification: D72, D78.

Keywords: Rational political budget cycles; time consistency; separation of powers; checks and balances; budgetary process.

1 Introduction

The standard approach to rational political budget cycles (PBC) assumes that a single fiscal authority has discretionary power over fiscal policy, and that under asymmetric information the executive can exploit this power for electoral purposes. However, in constitutional democracies, the process of drafting, revising, approving, implementing and controlling the budget requires the participation of the legislature. If the assumption of a single fiscal authority is dropped, the possibility of PBC will depend on the leeway that the legislature allows the executive in pursuing electoral destabilization (Streb 2005).

The Romer and Rosenthal (1978) and (1979) agenda setter model allows to model the interaction between the executive and the legislative powers in the fiscal process. Persson, Roland and Tabellini (1997) use this framework to analyze how separation of powers allows to control the rents of politicians. Saporiti and Streb (2004) use this to analyze PBC, with a legislature that
acts benevolently, as a representative of the interests of the people, finding
that the legislature may have a moderating role on electoral cycles.\footnote{Given asymmetric information on the budget, the executive is tempted to distort the composition of spending towards more visible public goods to increase its electoral chances, as in Rogoff (1990). With an exogenous status quo, the moderating role of the legislature depends on the status quo, on whether the executive or the legislature have the agenda-setting authority, and on the degree of compliance with the budget law.}

Our point here is that separation of power \textit{per se} is not enough to eliminate PBC. Separation of powers only has a bite in the fiscal process when the executive and legislative branches are not perfectly aligned. Our approach emphasizes the role of divided government in moderating electoral cycles. This draws on the insight of Alesina and Rosenthal (1995) on the moderating influence of an opposition legislature. Through the metric of veto players (Tsebelis 2002), this insight applies not only to divided government in presidential systems, but more generally to coalition governments. Coalition members start to compete among themselves for votes, so it is particularly hard for different political parties to collude close to elections.

We now sketch the relationship between separation of powers and aggregate PBC when there is asymmetric information on fiscal policy. The competence of the executive just before elections matters for performance after elections, so retrospective voting is rational (Rogoff and Sibert 1988). The executive does not know its competence, operating under uncertainty about the effect of its policy actions. This follows the timing in Lohmann (1998), who focuses on the credibility problems of economic policy in electoral periods, abstracting from the signaling dimension pointed out by Rogoff and Sibert (1988). In this framework, the executive will be tempted in electoral periods to increase expenditure and reduce taxes to increase its electoral chances. Additionally, debt financing is distortionary, so there is short run cycle where policies are reversed after elections, as in Shi and Svensson (2006) and Alt and Lassen (2006a, b).

With discretion, the ex-ante optimal fiscal policy is not credible ex-post, so the legislature may play a credibility role. The role of the legislature in preventing electoral destabilization turns out to be crucial, because of its veto power to reject new indebtedness. The authorization of new debt is a standard prerogative of the legislature in all budget rules. When the legislature is aligned with the executive, it will not curb cycles because it shares its same electoral objectives. On the other hand, if the legislature is not aligned with the executive, it will not be interested in increasing the chances of success of the executive, so it will veto these electoral changes in the budget. For this veto power to be effective in avoiding PBC, the legislature needs the oversight and enforcement capacity to insure that the executive complies with the approved budget law.

Consequently, to solve the credibility problem of fiscal policy around elections it is not enough to have a budget rule that prohibits issuing debt
for electoral purposes. To use Montesquieu’s words, it is necessary to have a power that checks power, in order to avoid the credibility problem of discretionality.

Section 2 describes the setup to study the role of divided government in PBC, and Section 3 analyzes the equilibria. Section 4 presents a possible extension of the basic setup to the case where divided government leads government competence to depend on the parties that share government. Section 5 presents the tentative conclusions and implications.

2 Checks and balances

2.1 Preferences

Consider an infinite-horizon society. Let $t$ denote time, where odd positive integers are electoral periods and even positive integers are non-electoral periods.

The society is composed by a large but finite number of identical individuals, labeled $i = 1, 2, \ldots, n$. There is a representative individual that cares about the competence of the incumbent in providing public goods. Following Alt and Lassen (2006a), we could alternatively assume there is heterogeneity among voters, in which case these preferences would represent instead the median voter. Under this alternative interpretation, our assumption implies that the median voter is indifferent between the incumbent and the opposition in terms of ideology, so in equilibrium its vote is determined by the expected competence of each.$^2$

In every period $t$, individual $i$ plays roles both as a consumer and as a citizen. The representative consumer derives utility from a public good $g_t$ and a private good $c_t$. The representative consumer’s per-period payoff is given by a quasi-linear utility function,

$$u(c_t, g_t) = c_t + \alpha \ln(g_t), \tag{1}$$

where $0 < \alpha < 1$. The intertemporal utility function $U$ is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, g_t), \quad 0 < \beta < 1. \tag{2}$$

Output $y_t$ is exogenous, with $y_t = y$. By the consumer’s per-period budget constraint, consumption $c_t$ equals disposable income, namely, $y$ net of tax payments $p_t$:

$$c_t = y - p_t. \tag{3}$$

$^2$If the median voter were more inclined towards one of the parties, this could be represented by an additional term that pulled preferences towards right or left, following the ideas in Alt and Lassen (2006a).
2.2 Government

Each period $t$, the government is subject to the budget constraint

$$\gamma_t = \pi_t + d_t - (1 + r)d_t - 1,$$  \hspace{1cm} (4)

where $\gamma_t$ denotes actual budget expenditures on public goods, $\pi_t$ are tax revenues, $d_t$ is public debt and $r$ is the interest rate on debt, that is constant.

Public resources $\gamma_t$ are transformed into the public good $g_t$ according to the competence $\theta_t$ of the government:

$$g_t = \theta_t \gamma_t.$$  \hspace{1cm} (5)

Similarly, the competence of the government affects how tax payments $p_t$ become actual government tax receipts $\pi_t$, reflecting, among other things, the use of more or less distortionary taxes:

$$p_t = \frac{\pi_t}{\theta_t}.$$  \hspace{1cm} (6)

By (5), to provide a given level of public goods, expenditure must be higher with less competent governments. Similarly by (6), to generate a given level of tax receipts, tax payments must be higher with less competent governments.

The representative individual (alternatively, the median voter, as mentioned above) cares about the competence of the incumbent in providing public goods. Since the incumbent does not know its competence when it takes budget decisions, from its viewpoint the electoral outcome is uncertain.\(^3\)

Our technological assumptions lead taxes and expenditures to fluctuate with the competence of the government. Since voters are inclined to reelect more competent incumbents, this creates an electoral incentive for governments to lower taxes in electoral years. It also creates an incentive to increase expenditure using debt finance. In contrast, in Shi and Svensson (2002) and Alt and Lassen (2006a, b) electoral cycles in the budget balance are exclusively through expenditure cycles, not tax and expenditure cycles.

We assume that the competence of the government depends on the competence of the party that controls the executive branch $E$. For each party $i = A, B$, competence is partially lasting, following a first-order moving average process as in Rogoff and Sibert (1988) and others (a subscript $i$ for each party is omitted here):

$$\theta_t = \tilde{\theta} + \varepsilon_{t-1} + \varepsilon_t.$$  \hspace{1cm} (7)

\(^3\)Given our timing, to have uncertain electoral outcomes it is not necessary to introduce probabilistic voting through a looks shock as in Rogoff (1990), or uncertainty about relative preferences for candidates as in Shi and Svensson (2006).
Each competence shock $\varepsilon$ is a random i.i.d. variable which is uniformly distributed over the interval $\left[-\frac{1}{\xi}, \frac{1}{\xi}\right]$, with expected value $E(\varepsilon) = 0$ and density function $\xi > 0$. A higher value of $\varepsilon$ corresponds to a more competent politician. The probability distribution of competence $\theta_t$ conditional on $\varepsilon_{t-1}$, $F(\theta_t|\varepsilon_{t-1})$, is also uniform, with support $\left[\bar{\theta} + \varepsilon_{t-1} - \frac{1}{\xi}, \bar{\theta} + \varepsilon_{t-1} + \frac{1}{\xi}\right]$, and $E(\theta_t|\varepsilon_{t-1}) = \bar{\theta} + \varepsilon_{t-1}$. Henceforth, $\bar{\theta} > 1/\xi$, so $\theta_t > 0$ and (5) and (6) are well-defined.

### 2.3 No debt condition

Following the insight in Shi and Svensson (2006), the quasilinear preferences in (1), jointly with an assumption about the value of the discount factor $\beta$ and the interest rates, can drastically simplify the optimal policy problem. Whereas Shi and Svensson (2006) assume the interest rate is increasing in the level of debt, we assume that the rate $r$ at which the government can borrow is constant, but this borrowing rate is larger than the rate $r'$ at which it can lend, and $r > r' > 0$.

Furthermore, we assume the following condition is satisfied, which will assure that neither debt nor holding financial assets will be optimal in equilibrium:

$$
\frac{1}{(1 + r)} \frac{E_t\left(\frac{1}{\theta_t}\right)}{E_t\left(\frac{1}{\theta_{t+1}}\right)} < \beta < \frac{1}{(1 + r')} \frac{E_t\left(\frac{1}{\theta_t}\right)}{E_t\left(\frac{1}{\theta_{t+1}}\right)}.
$$

(8)

Since $\frac{\bar{\theta} - \frac{1}{\xi}}{\bar{\theta} + \frac{1}{\xi}} < \frac{E_t\left(\frac{1}{\theta_t}\right)}{E_t\left(\frac{1}{\theta_{t+1}}\right)} < \frac{\bar{\theta} + \frac{1}{\xi}}{\bar{\theta} - \frac{1}{\xi}}$, a sufficient condition for (8) to hold is

$$
\frac{1}{(1 + r)} \frac{\bar{\theta} + \frac{1}{\xi}}{\bar{\theta} - \frac{1}{\xi}} < \beta < \frac{1}{(1 + r')} \frac{\bar{\theta} - \frac{1}{\xi}}{\bar{\theta} + \frac{1}{\xi}}.
$$

(9)

This condition assures that $\beta(1 + r) > 1 > \beta(1 + r')$, an assumption we maintain throughout. This implies that in expected value there is a loss of utility if debt is used to finance present consumption, or if asset accumulation is used to finance future consumption.

### 2.4 Asymmetric Information

The timing, as in Lohmann (1998a), is that in each period $t$ incumbents do not observe the value of $\varepsilon_t$ before making budget decisions. The interpretation of this timing is that policy is decided under uncertainty, so the choice of the policy instrument is not equivalent to the choice of an outcome, but rather to the choice of a lottery of outcomes.
Voters know the incentives political parties face and the objectives they try to achieve. Though the representative (median) voter knows the structure of the budget process, it does not observe either the executive party’s most recent competence shock, $\varepsilon_t$, or the budget decisions ($\gamma_t$, $\pi_t$, $d_t$) before voting. The only information it receives is the amount of public good $g_t$ that is provided, and of tax payments $p_t$ it makes. Thus, incumbents have a temporary information advantage over the actual budget allocation implemented. All past competence shocks are common knowledge.

2.5 Veto players

The agenda setter model of Romer and Rosenthal (1978, 1979) allows to reduce the policy-making process carried out to set the budget to the interaction of the current leaders of the two branches of government, the executive and the legislature.

The terms in office in the executive and legislative branches last two periods. Every other period, the electorate removes or confirms the executive and legislative leaders in an explicit electoral contest (we are abstracting from midterm legislative elections). If the incumbent is confirmed, it controls this branch for another term. Otherwise, the opposition takes office.

We assume there are two parties, $A$ and $B$. A party’s payoffs are as follows. Besides caring about the utility from the consumption of private and public goods, when a party wins executive elections and its leader becomes the $E$ incumbent, it receives an exogenous rent $\chi^E > 0$ at the beginning of each term in office. The party that wins legislative elections and controls $L$ receives a rent $\chi^L \geq 0$, where $\chi^L < \chi^E$. These rents reflect the strength of the electoral goal, to use Lohmann’s (1998a) words, and will be the source of conflict between political parties and the electorate.

Through the idea of veto players, the agenda setter model can be used to reflect not only presidential systems, but also the working of parliamentary systems (Tsebelis 2002). While in a presidential system, $E$ is the leader of the executive and $L$ is the leader of the legislature, in a parliamentary system $E$ can be taken to represent the leader of the majority coalition party, and $L$ the leader of the minority coalition party.

If $E$ and $L$ are controlled by the same party, there is no veto player: in a presidential system, this is referred to as unified government, when the executive has an aligned legislature; in a parliamentary system, as single-party rule where one party has a majority of seats in the legislature. There are veto players in a presidential system when there is divided government, and the legislature is controlled by an opposition party whose electoral motives are strictly opposed to those of the executive; in a parliamentary system, something similar happens when the party that leads government is forced to form a coalition to reach a majority of seats in parliament.
What does not translate so easily from a presidential system to a parliamentary system is the voting decision. In a presidential system, each voter has two separate votes, and can decide whether to support the same party in the executive branch and the legislature. In a parliamentary system, an individual voter cannot literally split its vote among two political parties, since there is no separate vote for the executive. However, the representative voter has a preference for whether it wants a single-party government, or whether it wants a coalition government. If we allow for fictitious vote splitting, so the voter can distribute its vote in a given proportion between parties A and B, this can artificially recreate what the electorate at large can do. With our representative voter who can split votes, we are skipping over the need to coordinate votes among the electorate at large, and the specific process by which certain vote totals lead either to a single-party or to a coalition government. Our purpose at hand is to see the consequences for PBC of whether only one or more than one party run the government.

### 2.6 Budget process

$E$ makes a budget allocation proposal, which must be accepted by $L$ to become law. No amendment rights exist, so $L$ faces a take-it-or-leave-it proposal where the reversion outcome (the status quo) in case of rejection is specified below. This is the case where $E$ has most power; we later review the case where $L$ can amend $E$’s proposal, so $L$ has the agenda setting power. The proposals are in terms of budget expenditure and debt, because the budget restriction determines the required tax revenues (only two of these three variables can be chosen freely).

- The timing of the budget process in period $t$ is as follows:

1. $E$ proposes $\hat{\gamma}_t^E$, $\hat{d}_t^E$ to $L$.

2. Since $L$ has no amendment rights, $L$ chooses whether to accept the proposal or not. If the proposal is not accepted, the budget is given by status quo $\bar{\gamma}_t$, $\bar{d}_t$. This will determine the approved budget $\tilde{\gamma}_t$, $\tilde{d}_t$.

3. $E$ implements $\gamma_t$, $d_t$, which equals the approved budget under perfect compliance.

4. $\varepsilon_t$ is realized and $g_t$ and $p_t$ are determined according to (5) and (6);

5. Voters observes $g_t$ and $p_t$, but not $\varepsilon_t$ nor $(\gamma_t, \pi_t, d_t)$, forming a belief $\hat{\theta}_t$ about the incumbent’s competency.

6. Without loss of generality, we assume party $A$ controls $E$. If $t$ is an odd positive integer, i.e., an electoral period, voters decide whether to reelect party $A$ in $E$, and whether to vote incumbent party $A$ or opposition party $B$ for $L$. 

7
7. Individuals observe $\varepsilon_t$ and $(\gamma_t, \pi_t, d_t)$ and period $t$ ends.

2.7 Budget rule

As is standard in the agenda setting model, if the executive’s budget proposal is rejected, the status quo for expenditure is given by an exogenous reversion outcome:

$$\tilde{\gamma}_t = \bar{\gamma}.$$ (10)

Though taxes must be authorized by the legislature, since the executive will have no incentive in equilibrium to tax beyond expenditure (or, if there is outstanding debt, beyond expenditure plus the amount needed to rescue outstanding debt), it is not necessary to explicitly introduce any additional restriction on this front.

As to the status quo for debt, we assume that there is an endogenous debt ceiling:

$$d_t \leq d_{t-1}.$$ (11)

This endogenous debt ceiling merely reflects the restriction that, unless authorized by $L$, the outstanding amount of debt cannot be increased. This budget rule is typical of budget processes.

3 Equilibrium

As a benchmark, we first study the case when there are no elections. We then analyze elections under two setups, when $E$ has full discretionary power, and when there is divided government so the opposition party controls $L$ and can check the power of $E$.

3.1 No elections

A candidate is randomly selected in period $t = 0$, and remains in office forever. By quasilinear preferences, the marginal utility of consumption is equal to one. If, in expected value, the marginal utility of the public good is equal to the marginal utility of consumption, any extra resources the government may have will be optimally used to reduce taxes.

Suppose the government resorts to an extra dollar of debt in period $t$ to reduce taxes. From expressions (1), (2), (3) and (6), it follows that expected utility increases $E_t \left( \frac{1}{\theta_t} \right)$ in period $t$. If the extra dollar of debt is repaid next period, utility falls by $(1 + r)E_t \left( \frac{1}{\theta_{t+1}} \right)$ in period $t + 1$. Since the future is
discounted at the rate $\beta$, it will never be optimal to borrow an extra dollar and repay it in the next period, because by (8):

$$\beta(1 + r)E_t\left(\frac{1}{\theta_{t+1}}\right) > E_t\left(\frac{1}{\theta_t}\right).$$

Here $E_t\left(\frac{1}{\theta_{t+1}}\right)$ equals unconditional expectation, since there is no information on current shock when decision is taken.\footnote{This condition also rules out that repaying the debt farther out in the future is optimal, because $(1 + r) > 1$, so the compounding effect makes the condition more binding for $t \geq t + 2$.} Following an analogous argument, condition (8) also rules out the possibility that the government may become a net lender. This leads to a corner solution with no debt nor financial assets.

Since our assumptions about $\beta$, $r$ and $r'$ in (8) assure that $d_t = 0$ (i.e., $\gamma_t = \pi_t$) for $t = 0, 1, ..., \ldots$, the intertemporal problem can be broken in a sequence of simpler optimization problems:

$$\text{Max } E_t[c_t + \alpha \ln(g_t)]$$

$$\{\gamma_t, \pi_t\}$$

s.t. (3), (4), (5) and (6).

The solution, using the properties of the uniform distribution, and then integrating, is:

**Proposition 1** Without elections, the executive will choose optimal expenditure and tax collection each period:

$$\gamma_t^* = \pi_t^* = \frac{\alpha}{E_t[\frac{1}{\theta_t}]} = \frac{\alpha}{\xi \ln(\frac{\theta + \epsilon_{t-1} + \frac{1}{\theta_t}}{\theta + \frac{1}{\theta_t}})}, \ t = 0, 1, \ldots$$

(12)

Fiscal policy $\gamma_t^*$ and $\pi_t^*$ depends on expected competence; differentiation of (12) shows both are increasing in the past competence shock $\epsilon_{t-1}$:

$$\frac{\partial \gamma_t^*}{\partial \epsilon_{t-1}} = \frac{\alpha}{[E_t[\frac{1}{\theta_t}]]^2}E_t[\frac{1}{\theta_t^2}] > 0$$

(13)

The expected provision of the public good is increasing in the past competence shock for two reasons, higher expected competence and a larger budget for the public good:

$$\frac{\partial E_t[g_t]}{\partial \epsilon_{t-1}} = \frac{\partial E_t[\theta_t \gamma_t^*]}{\partial \epsilon_{t-1}} = \gamma_t^* + E_t[\theta_t] \frac{\partial \gamma_t^*}{\partial \epsilon_{t-1}} > 0$$

(14)
Though taxes are increasing in the past competence shock, expected consumption of the private good is constant, since the increase in legislated taxes is exactly compensated by larger efficiency in tax collection:

\[
E_t[c_t] = E_t[y - p_t] = y - E_t[\frac{\pi^*_t}{\theta_t}] = y - \pi^*_t E_t[\frac{1}{\theta_t}] = y - \alpha. \tag{15}
\]

Since the budget is decided ex-ante, it cannot be conditioned on the current competence shock \(\varepsilon_t\). Ex-post, a more competent incumbent generates a greater provision of the public good with a given expenditure and imposes a lower burden on tax payers, lowering the relative cost of public versus private goods.

### 3.2 Unchecked executive

Consider next the model with regular elections every two periods. There is only one policy-maker, the executive. Odd integers are electoral periods and even integers are non-electoral years.

The players are the incumbent party \(A\), the opposition party \(B\), the representative (median) voter \(V\), and Nature. From the viewpoint of the representative (median) voter \(V\), the two parties only differ in competence. Because the competence shocks are transitory, each election can be treated separately, so the infinite-horizon model can be broken down into a series of separate problems. Using backwards induction, the solution can be found in a sequence of steps.

**Step 1: The incumbent’s decision in a nonelectoral period**

In period \(t + 1\), a nonelectoral period, the incumbent (either \(A\) or \(B\)) has no incentive to manipulate the voters’ perception of its competence, since the outcome of future elections will depend on the expected competence in \(t + 3\), which is uncorrelated with competence in \(t + 1\). Since the optimal strategies of all incumbents in the post-electoral period are the same, the distinction between the original and the potential incumbents is omitted to simplify the notation. Hence:

\[
\gamma_{t+1}^{ue} = \gamma^*_t + \frac{\alpha}{E_{t+1}[\frac{1}{\theta_{t+1}}]}, \tag{16}
\]

\[
\pi_{t+1}^{ue} = \gamma^*_t + (1 + r)d_t, \tag{17}
\]

where the superscript \(ue\) refers to unchecked executive. In a nonelectoral period, the expenditure is the same as in a setup without elections, but there may be more taxes if the incumbent has to pay off the debt incurred in the last election period.

**Step 2: The government’s plausibility restriction**
At election time, voters observe $g_t, p_t$, but not $d_t, \gamma_t$ and $\pi_t$. Voters know that consumers and the government are subject to restrictions (3), (4), (5), (6).

There is an additional restriction, that we label a “plausibility” restriction. If the incumbent relies on debt, it must preserve the ratio between expenditures on public goods and tax collection that would obtain without electoral manipulation of the budget, to avoid making that action transparent to voters. This restricts the way the incumbent can use debt, forcing it to split it in specific proportions between more expenditure and less taxes.

Let $\pi_t^* = \gamma_t^*$ denote the budget that is not affected by opportunistic concerns, the optimal budget choice when there is no previous debt. To derive the observed levels of $g_t$ and $p_t$ that satisfy our plausibility restriction, let the actual budget choices $\gamma_t$ and $\pi_t$ be determined by a scale factor $\omega_t$ that takes into account the values of $g_t$ and $p_t$ possible under technological restrictions (5) and (6):

$$\gamma_t = \omega_t \gamma_t^*, \quad \pi_t = \frac{\pi_t^*}{\omega_t}. \quad (18)$$

This plausibility restriction implies that

$$\frac{\gamma_t}{\pi_t} = \omega_t^2.$$ 

That is to say, the budget items have to satisfy a certain ratio to replicate the distribution of outcomes without electoral manipulation, with the expected value of the distribution shifted to the right by $\omega_t > 1$ (the government is tempted to mimic positive competence shocks, not negative ones). This restriction implies that, beyond identity (4), debt must also satisfy:

$$d_t = \gamma_t - \pi_t = \left(\omega_t^2 - 1\right) \pi_t = \left(\omega_t - \frac{1}{\omega_t}\right) \pi_t^*.$$ 

This plausibility restriction makes it clear that debt must be used in predefined proportions to reduce taxes and increase expenditures, to preserve the characteristics of the original distribution of competency shocks.

Voters know this restriction on government actions and include it in their estimation of the incumbent’s competence. If voters could observe $\omega_t$, they could easy calculate $\theta_t$, since $\frac{\omega_t}{\pi_t} = \frac{\gamma_t}{\pi_t} \theta_t^2 = \omega_t^2 \theta_t^2$, which implies that:

$$\theta_t = \sqrt{\frac{\omega_t}{\pi_t}}.$$ 

However, voters do not really observe $\omega_t$. They must estimate it, in order to estimate $\theta_t$. Therefore, if we call $\hat{\omega}_t$ voters’ estimate of $\omega_t$, the estimate $\hat{\theta}_t$ of $\theta_t$ is:

$$\hat{\theta}_t = \sqrt{\frac{\hat{\omega}_t}{\hat{\pi}_t}}.$$
Using this expression, voters can estimate the incumbent’s current competence shock ($\varepsilon_{t-1}$ is already known in period $t$):

$$\hat{\varepsilon}_t = \hat{\theta}_t - \bar{\theta} - \varepsilon_{t-1} = \sqrt{\frac{p_t}{\omega_t}} - \bar{\theta} - \varepsilon_{t-1}. \quad (19)$$

**Step 3: The citizen’s vote**

Voters compare the expected utility next period if they vote either the incumbent or the challenger. Voters can estimate the competence shock of the incumbent, but nothing can be concluded about the opposition from the observed policy actions of the government.

In regard to the opposition, voters only know the distribution of $\varepsilon_t$ and hence that $E_t[\varepsilon_t] = 0$. Hence, expected utility from a vote for the opposition is unconditional:

$$E_t[c_{t+1} + \alpha \ln(g_{t+1})] = E_t[y - \frac{\pi^{ue}_{t+1}}{\theta_{t+1}} + \alpha \ln(\theta_{t+1} \gamma^{ue}_{t+1})] \quad (20)$$

Expected utility from a vote for the incumbent is conditional on the current competence shock:

$$E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \hat{\varepsilon}_t] = E_t[y - \frac{\pi^{ue}_{t+1}}{\theta_{t+1}} + \alpha \ln(\theta_{t+1} \gamma^{ue}_{t+1}) \mid \hat{\varepsilon}_t] \quad (21)$$

In order to determine voters’ decisions, we must compare these two expressions. We formally do this comparison in the following proposition. The Appendix shows that $E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$ is increasing and concave in $\varepsilon_t$.

**Proposition 2** Given $E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$ increasing and concave in $\varepsilon_t$, $E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \hat{\varepsilon}_t] \geq E_t[c_{t+1} + \alpha \ln(g_{t+1})]$ if and only if $\hat{\varepsilon}_t \geq 0$.

**Corollary 1** Voters vote for the incumbent if and only if $\hat{\varepsilon}_t \geq 0$.

**Proof 1** For a proof of the proposition please see the Appendix. The proof of the corollary is immediate from (20) and (21).

We now employ this proposition to compute the probability that the incumbent wins the election. Let’s call this probability $\mu_t = \Pr(\hat{\varepsilon}_t > 0) = \Pr\left(\frac{\sqrt{p_t}}{\omega_t} - \bar{\theta} - \varepsilon_{t-1} > 0\right)$. Considering that the actual value of $\varepsilon_t$ equals $\frac{\sqrt{p_t}}{\omega_t} - \bar{\theta} - \varepsilon_{t-1}$, adding these terms to each side and simplifying, we get $\mu_t = \Pr\left[\varepsilon_t > \frac{\sqrt{p_t}}{p_t} \left(\frac{1}{\omega_t} - \frac{1}{\omega_t}\right)\right]$. Finally, considering that $\varepsilon_t$ follows a uniform distribution with density $\xi$, we obtain:

$$\mu_t = \Pr(\hat{\varepsilon}_t > 0) = \frac{1}{2} + \xi \sqrt{\frac{p_t}{p_t}} \left(\frac{1}{\omega_t} - \frac{1}{\omega_t}\right). \quad (22)$$
Notice that if voters are surprised ($\omega_t > \hat{\omega}_t$) the incumbent increases its probability of winning above the value $\frac{1}{2}$. Furthermore, note that:

$$\frac{\partial \mu_t}{\partial \omega_t} = \xi \sqrt{\frac{g_t}{p_t} \frac{1}{\omega_t^2}} > 0.$$  \hspace{1cm} (23)

**Step 4: The incumbent’s decision in an electoral period**

Taking into account $\mu_t$, the endogenous probability that the incumbent is reelected, the incumbent’s objective function is:

$$\max_{\{\gamma_t, \pi_t, d_t\}} E_t[c_t + \alpha \ln(g_t) + \beta(c_{t+1} + \alpha \ln(g_{t+1})) + \beta \mu_t \chi^E]$$

$$\text{s.t. (3), (4), (5), (6), (18) and (22).}$$

Incorporating these restrictions, the government’s problem in the electoral period can be reframed in terms of the choice of $\omega_t$, that will determine all fiscal variables:

$$\max_{\omega_t \geq 1} E_t \left\{ y - \frac{\pi_t^*}{\omega_t} + \alpha \ln(\theta_t + \ln \pi_t^* + \ln \omega_t) \right. \left. + \beta \left[ y - \frac{\gamma_t^* + (1 + r) \pi_t^*}{\theta_{t+1}} \left( \omega_t - \frac{1}{\omega_t} \right) + \alpha \ln(\theta_{t+1} + \ln \gamma_{t+1}) \right] + \beta \left[ \frac{1}{2} + \xi \theta_t \left( \frac{\omega_t}{\omega_t^*} - 1 \right) \right] \chi^E \right\}$$

The first order condition is given by:

$$\frac{dE_t[\cdot]}{d\omega_t} = E_t \left[ \frac{\pi_t^*}{\theta_t} \frac{1}{\omega_t^2} + \frac{\alpha}{\omega_t} - \frac{\beta(1 + r) \pi_t^*}{\theta_{t+1}} \left( 1 + \frac{1}{\omega_t^2} \right) + \beta \xi \frac{\theta_t}{\omega_t} \chi^E \right] \leq 0,$$

with strict equality if $\omega_t > 1$

which can be simplified, using the definition of $\pi_t^*$:

$$\frac{dE_t[\cdot]}{d\omega_t} = \frac{\alpha}{\omega_t^2} + \frac{\alpha}{\omega_t} - \beta(1 + r) \frac{\alpha}{E_t \frac{1}{\theta_{t+1}}} \left( 1 + \frac{1}{\omega_t^2} \right) + \beta \xi \frac{\theta_t}{\omega_t} \chi^E \leq 0,$$

with strict equality if $\omega_t > 1$.  \hspace{1cm} (24)
Note that $\frac{d^2 E_t}{d \omega_t^2} = -\frac{2\alpha}{\omega_t^2} + \frac{\alpha}{\omega_t^2} + 2\alpha \beta (1 + r) \frac{E_t(\frac{1}{\beta_{t+1}})}{E_t(\frac{1}{\sigma_t})} \frac{1}{\omega_t^2}$, which is strictly negative for $\omega_t \geq 1$ if the following condition holds:

$$
\beta (1 + r) \frac{E_t(\frac{1}{\beta_{t+1}})}{E_t(\frac{1}{\sigma_t})} < \frac{3}{2}.
$$

(25)

Assuming (25), the first order condition (24) becomes sufficient for an optimum.

People have rational expectations, so in equilibrium $\hat{\omega}_t$ must be equal to $\omega_t$. Therefore, if we call $\omega_{ue}$ the equilibrium value of $\omega_t$, we obtain:

$$
dE_t[\cdot] = \frac{\alpha}{(\omega_{ue}^{ue})^2} + \frac{\alpha}{\omega_{ue}^{ue}} - \beta (1 + r) \alpha \frac{E_t(\frac{1}{\beta_{t+1}})}{E_t(\frac{1}{\sigma_t})} \left( 1 + \frac{1}{(\omega_{ue}^{ue})^2} \right) + \beta \xi \frac{\theta + \epsilon_{t-1}}{\omega_{ue}^{ue}} \chi^E \leq 0,
$$

(26)

with strict equality if $\omega_{ue}^{ue} > 1$.

(27)

It is clear that if there is no opportunistic motive ($\chi^E = 0$), then this expression evaluated at $\omega_{ue}^{ue} = 1$ is negative (recall condition (8)) and the incumbent will not distort fiscal outcomes. On the other hand, with positive exogenous rents from power ($\chi^E > 0$) the above expression can became positive at $\omega_{ue}^{ue} = 1$, which implies that the incumbent prefers a strictly positive $\omega_t$. We summarize these results in the following proposition.

**Proposition 3** With an unchecked executive, assume that conditions (8) and (25) hold, i.e., $1 < \beta (1 + r) \frac{E_t(\frac{1}{\beta_{t+1}})}{E_t(\frac{1}{\sigma_t})} < \frac{3}{2}$. Let $\bar{\chi}_t = \frac{2\alpha}{\beta (1+r) \frac{E_t(\frac{1}{\beta_{t+1}})}{E_t(\frac{1}{\sigma_t})} - 1}$.

Then in an electoral period (t odd):

1. If $\chi^E \leq \bar{\chi}_t$ an unchecked executive does not distort fiscal outcomes ($\omega_{ue}^{ue} = 1$)
2. If $\chi^E > \bar{\chi}_t$ an unchecked executive distorts fiscal outcomes ($\omega_{ue}^{ue} > 1$).

**Corollary 2** Employing the expression (18) we obtain in an electoral period (t odd):

1. If $\chi^E \leq \bar{\chi}_t$ the $\gamma_{t}^{ue} = \gamma_{t}^*$ and $\pi_{t}^{ue} = \pi_{t}^*$
2. If $\chi^E > \bar{\chi}_t$ then $\gamma_{t}^{ue} = \omega_{t}^{ue} \gamma_{t}^*$ and $\pi_{t}^{ue} = \frac{\pi_{t}^*}{\omega_{t}^*}$
Since politicians are typically interested in being reelected, the natural assumption in an opportunistic framework is that $\chi^E > \bar{\chi}_t$, so the executive is willing to distort fiscal outcomes to be reelected.

Suppose that an unconstrained executive $E$ must formulate optimal plans in the initial non-electoral period $t = 0$. Viewed at $t = 0$, when the government sets policy in advance, the probabilities of reelection $\mu_t$ are exogenous and equal to $1/2$ in expected value. Therefore, the government’s best policy is to plan to pick $\gamma_t^*$ and $\pi_t^*$, that is socially optimal every period since it maximizes social welfare. The problem with this optimal plan, of course, is that it is not time-consistent: when an electoral period arrives, the government has an incentive to increase expenditure and reduce taxes. This credibility problem underlies Proposition 3 under an unchecked executive.

What happens if the status quo is set according to rule (11)? Well, if the rule were binding, this would effectively curb the credibility problem: in an electoral period the executive would prefer to use debt to increase expenditures and reduce taxes in order to look more competent, but the status quo rules out public indebtedness. However, it does not make sense to assume that the executive is constrained to follow any rule, unless it has to share the power to change rules with another body. Otherwise, if the executive is also vested with legislative power, it can do and undo any rule it likes, being effectively unconstrained. The natural environment where the executive shares rule-making power is when there is divided government, and an agreement has to be reached with the veto player $L$ on changes in the budget.

3.3 Separation of powers

We know turn to fiscal policy under separation of powers. We distinguish between divided and unified government. For both presidential and parliamentary systems, we describe divided government in terms of $E$ being in the hands of one party and $L$ in the hands of the other.\footnote{Saporiti and Streb (2004) consider separation of powers with a randomly elected legislature that represents the interests of the people, so the legislature is never aligned with the executive. Here, the issue of unified or divided government is endogenous and depends on voters.}

Suppose that in period $t = 0$ the randomly selected government is a unified one. Without loss of generality, assume that party $A$ control both the executive and the legislature. Let debt $d_{-1} \geq 0$. Since we have an aligned legislature or a single party government, nobody will veto proposals by $E$. This implies, by Proposition 3, that with sufficiently large opportunism there will be an electoral cycle in fiscal policy in $t = 1$, the first electoral period. In period $t = 0$ party $A$ does not have any incentive to distort fiscal variables. Hence, it just selects optimal expenditures and repays past debt, if any.
As regards voters, at election in $t = 1$ they will want the party with the highest expected competency holding the executive, just as in the case of an unchecked executive. At the same time, we conjecture they will want to have divided government. This is so, because in terms of government competence it is indifferent for voters whether a single party controls both the executive and the legislature, or if two parties share control. But in terms of the distortion of fiscal variables, divided government is strictly preferred if an opposition legislature can block the executive’s attempts to distort the budget in period $t = 3$ to look more competent. Putting all this together, we deduce that voters will prefer to vote for divided government in period $t = 1$.

Does what actually happens in periods $t = 2$ and $t = 3$ under divided government conform to these conjectures? Let us assume that $A$ controls the executive and $B$ the legislature. In the electoral period $t = 3$, the executive would like to increase its electoral chance by using debt to select $\pi_{T+1}^e$ and $\gamma_{T+1}^e$. However, party $B$ can veto this and any attempt of $A$ to employ debt to increase expenditures and reduce taxes, since the status quo debt restriction given by (11), i.e., $D_3 \leq D_2$, introduces an effective restriction in the executive’s opportunities. Party $B$ has the motivation and the power to veto any attempt of party $A$ to use debt to increase its electoral chances. Therefore, party $A$ is forced to set expenditures equal to taxes. Given that it cannot affect its perceived competency, the best party $A$ can do is to select an optimal level of taxes and expenditures. Notice also that, should the legislature veto these optimal level of taxes and expenditures, this would not affect the voters’ perception of party $A$’s competence, since what voters use in their inference problem is the no new debt restriction, which implies that $\gamma_3 = \pi_3$, so the ratio $g_t/p_t$ can be used to infer competency whatever the level of expenditure. Given this, the legislature has no incentive to block optimal expenditure in election periods.

As to the non-electoral period $t = 2$, the executive, controlled by party $A$, chooses an optimal expenditure and repays past debt, if any, because whatever it does then does not affect its electoral chances in the next electoral period, only current welfare. The legislature, controlled by party $B$, does not want to veto this proposal, because if does not affect future reelection prospects of either party, and it leads to optimal outcome in non-electoral period. This confirms the voters’s conjectures we assumed at the outset.

Putting together the arguments of the last three paragraphs, and extending the logic to all future time periods, we get the following conclusion:

**Proposition 4** Suppose there is separation of powers and the legislature must authorize new debt. Under perfect compliance with the budget law, there are no electoral cycles with divided government. There are only electoral cycles with unified government.
If the only role of the veto player is to prevent electoral manipulations of fiscal policy, voters would always prefer divided government. In the next section, we consider what happens when shared government implies a trade-off between competence and stability.

What happens when the legislature has amendment powers? The result is unchanged. Since $L$ can prevent new indebtedness in net terms, then an unaligned legislature would not be willing to authorize the use of debt for electoral purposes, so $d_t = d_{t-1}$. At the same time, $L$ will be willing to authorize the optimal level of expenditure $\gamma^*_t = \pi^*_t$, since a lower level of expenditures and taxes does not reduce $E$’s reelection chances, since voters use the ratio $g_t/p_t$ to infer competency.

3.4 Imperfect compliance

At the implementation stage, $E$ supplies the public goods, but it is monitored by $L$. We now introduce a measure of the effective compliance with the balanced budget rule. Either there is perfect compliance, or imperfect compliance: $\lambda \in \{0, 1\}$. The measure $\lambda$ can be interpreted as the degree of compliance with the authorized budget, and describes the effective limits $L$ imposes on the executive office.

Proof 1 Proposition 5 Suppose there is separation of powers and the legislature must authorize new debt. If there is no compliance with the budget law, divided government cannot check the electoral manipulation of the budget.

This proposition implies a sharp (and falsifiable) prediction: PBC should be present in countries with imperfect compliance with the law. These countries should also have less incidence of divided government. This can be empirically related to the evidence on the existence of stronger cycles in developing countries, where there is typically less compliance with the rule of law than in developed countries. Streb, Lema, and Torrens (2007) study these empirical implications.

If there is no compliance with the budget law, divided government is useless. Hence, there is no incentive to have divided government. The model leads to expect more divided government in countries where there is compliance with the budget law, because here is where divided government actually puts a break on PBC. The evidence indeed points in that direction, since countries with higher rule of law (a general proxy of the respect for laws) have an average more veto players. See Table 5 in Streb, Lema, and Torrens (2007).
4 Shared government

The agenda setter model of Romer and Rosenthal (1978, 1979), where the policy-making process carried out to set the budget can be reduced to the interaction of the current leaders of the two branches of government, the executive and the legislature, can be combined with a stylized model of government performance when parties share power.

Suppose that, under divided government, the competence shocks of the government are a proportion \( \rho \in (1/2, 1] \) of the competence shocks of the party in charge of the executive:

\[
\theta_t = \rho \theta^i_t, \quad \text{where} \quad i \in \{A, B\}. \tag{28}
\]

This allows to capture the fact that divided government involves a trade-off between moderation of the executive, and efficiency. This trade-off depends on the competence of the party in charge of running the executive, because divided government is a drag on efficiency when there are positive competence shocks.

5 Conclusions

The fact that the executive incumbent is unable to credibly commit to not use debt for electoral purposes is at the heart of aggregate PBC (Shi and Svensson 2006, Alt and Lassen 2006a,b). This problem is generated by the discretionary power of the executive. Instead, when there is separation of powers, appropriate checks and balances work as a commitment device that reduces the size of electoral fiscal cycles, making all players better off (including the executive incumbent).

More generally, in relation to the debate on rules versus discretion, our discussion of PBC shows that a way to solve the credibility problem, making the budget rule a credible commitment, is to introduce an institutional arrangement of separation of powers that limits the discretion to change rules. However, the actual checks and balances under separation of powers are endogenous, since they depend on whether voters pick a unified or divided government.

For the United States, Alesina, Roubini, and Cohen (1997, chaps. 4 and 6) link the lack of recent evidence on opportunistic cycles to the fact that after 1980 many federal transfer programs became mandatory by acts of Congress, so they cannot be easily manipulated for short run purposes. Nordhaus (1989) traces the roots of this move towards less discretionary spending back to the Nixon administration, which prompted the US Congress to establish in 1974 the Congressional Budget Office to have an independent control of the budget.

From this model, one can derive a sharp empirical implication for aggregate PBC: if there is perfect compliance with the budget law, the budget rule
is credible if the party of the executive’s leader does not control the legislature. On the other hand, if there is imperfect compliance, the budget rule is never credible. Consequently, PBC should be larger either in countries with low legislative checks and balances, or with low observance of the rule of law. Streb, Lema and Torrens (2007) empirically study this conjecture on the role of divided government in political budget cycles (PBC). With a country panel of democracies over the 1960-2001 period, they find that legislative checks and balances indeed moderate cycles. When the discretional component of executive power is isolated, there are significant cycles in all types of democracies, including established democracies in OECD countries. Schuknecht (1996) had already suggested that stronger PBC in developing countries might be due to the existence of weaker checks and balances there.

6 Appendix: Proof of Proposition 1

We will prove this proposition in two steps. First, we will prove that the expected value of a function of two stochastic independent variables is equal or greater than the expected value of the same function, conditional on the realization of a third variable that generates an estimation of one of the variables, if and only if the function is increasing and concave. Secondly, we will prove that $u(c, g)$, considered as a function of the two independent stochastic variables $\varepsilon_t$ and $\varepsilon_{t-1}$ is increasing and concave.

Lemma 1 Let $Z = h(X, Y)$ be a function of two independent stochastic variables $X$ and $Y$, with marginal densities $f_x(x)$ and $f_y(y)$. Let us call $g(x) = E[Z \mid x]$ the expected value of $Z$ conditional on $x$. Consider a known vector of information variables $W$ that allows to estimate $X$ and call $\hat{x}(w)$ the calculated value of $X$ when $W$ adopts the value $w$. Suppose that $g(x)$ is an increasing and concave function of $x$. Then

$$E[Z \mid \hat{x}(w)] \geq E[Z] \text{ if and only if } \hat{x}(w) \geq E[X].$$

Proof 2 First notice that since $X$ and $Y$ are independent stochastic variables, $g(x) = E[Z \mid x] = \int h(x, y) f_y(y) \, dy$. Since $g(x)$ is concave, by Jensen’s inequality it follows that $g(E[X]) \geq E[g(x)]$. Employing the definition of $g$, the first term of the inequality is $E[Z \mid E[X]]$, while the second term equals $E_X[E[Z \mid X]]$. Therefore, $E[Z \mid E[X]] \geq E_X[E[Z \mid X]]$. By the law of iterated expectations, $E[Z] = E_X[E[Z \mid X]]$. Hence,

$$E[Z \mid E[X]] \geq E[Z]. \quad (29)$$

Now, consider the vector of information variables $W$, whose realization $w$ is known. It is clear from inspection of (29) that if $g(x) = E[Z \mid x]$ is an increasing function of $x$, then $E[Z \mid \hat{x}(w)] \geq E[Z \mid E[X]]$ if and only if $\hat{x}(w) \geq E[X]$. Therefore, $E[Z \mid \hat{x}(w)] \geq E[Z]$ if and only if $\hat{x}(w) \geq E[X]$.
In our case the two stochastic independent variables are $\varepsilon_t$ and $\varepsilon_{t+1}$, the vector of information variables is integrated by $g_t$, $p_t$, and $\hat{w}_t$, and $h(\varepsilon_t, \varepsilon_{t+1}) = c_{t+1} + \alpha \ln(g_{t+1})$. It remains to prove that $E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$ is an increasing and concave function.

We begin using expressions ((3), (4), (5), (6)) to replace $c_{t+1}$ and $g_{t+1}$ (line 1). Next we replace $\gamma_{t+1}^{ue}$ and $\pi_{t+1}^{ue}$ for their respective values (line 2). Finally, in line 3 we apply the conditional expected value operator.

$$E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t] = E_t[y - \frac{\alpha}{\theta_{t+1}} + (1 + r)d_t + \alpha \ln\left(\frac{\theta_{t+1} \alpha}{E_{t+1}[\frac{1}{g_{t+1}}]}\right) \mid \varepsilon_t]$$

$$= y - \alpha - (1 + r)d_tE_t[\frac{1}{\theta_{t+1}} \mid \varepsilon_t] + \alpha E_t[\ln\left(\frac{\theta_{t+1} \alpha}{E_{t+1}[\frac{1}{g_{t+1}}]}\right) \mid \varepsilon_t]$$

Note that $E_t[\frac{1}{\theta_{t+1}} \mid \varepsilon_t] = E_{t+1}[\frac{1}{g_{t+1}}]$ is used to simplify the expression for expected utility.

Expected utility is increasing in $\varepsilon_t$, a fact that can be confirmed by derivation:

$$\frac{\partial E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t} = \frac{\partial E_t[c_{t+1} \mid \varepsilon_t]}{\partial \varepsilon_t} + \alpha \left(E_t\left[\frac{1}{g_{t+1}} \frac{\partial g_{t+1}}{\partial \varepsilon_t} \mid \varepsilon_t\right]\right)$$

$$= -(1 + r)d_t \frac{\partial E_t[\frac{1}{\theta_{t+1}} \mid \varepsilon_t]}{\partial \varepsilon_t} + \alpha E_t \left[\frac{1}{\theta_{t+1} \gamma_{t+1}^{ue}} \left(\gamma_{t+1}^{ue} + \theta_{t+1} \frac{\partial \gamma_{t+1}^{ue}}{\partial \varepsilon_t}\right) \mid \varepsilon_t\right]$$

$$= (1 + r)d_tE_t[\frac{1}{\theta_{t+1}^2} \mid \varepsilon_t] + \alpha \left\{E_t\left[\frac{1}{\theta_{t+1}^2} \mid \varepsilon_t\right] + \frac{E_{t+1}[\frac{1}{g_{t+1}^2}] - E_{t+1}[\frac{1}{g_{t+1}}]}{E_{t+1}[\frac{1}{g_{t+1}}]}\right\} > 0$$

Expected utility in $t + 1$ is increasing in $\varepsilon_t$ because of three effects, represented by each term: a lower expected burden of outstanding debt, a higher expected competence in the provision of the public good, and a higher expenditure on the public good. Though future expenditure on the public good increases, expected consumption of the private good remains constant at $y - \alpha$ (see Proposition 1).

As to the second derivative of $E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$,

$$\frac{\partial^2 E_t[c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t^2} = \frac{\partial^2 E_t[c_{t+1} \mid \varepsilon_t]}{\partial \varepsilon_t^2} + \frac{\partial^2 E_t[\ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t^2}$$

The second derivative of the private consumption good is negative:
\[
\frac{\partial^2 E_t | \epsilon_{t+1} \mid \epsilon_t}{\partial \epsilon_t^2} = -2(1 + r)d_t \frac{1}{\theta_{t+1}^2} | \epsilon_t = -\frac{2(1 + r)d_t (\bar{\theta} + \epsilon_t)}{[(\bar{\theta} + \epsilon_t)^2 - (\frac{1}{\theta})]^2} < 0.
\]

As to the second derivative of the public good,

\[
\frac{\partial^2 E_t [\alpha \ln(g_{t+1})] | \epsilon_t}{\partial \epsilon_t^2} = \alpha \frac{\partial}{\partial \epsilon_t} \left( E_t \left[ \frac{1}{\theta_{t+1}} | \epsilon_t \right] + \frac{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]}{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]} \right) \\
= \alpha \left( -E_t \left[ \frac{1}{\theta_{t+1}^2} | \epsilon_t \right] - \frac{2E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]}{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]} + \frac{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2}{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2} \right).
\]

This expression can be simplified, since \( E_t \left[ \frac{1}{\theta_{t+1}} | \epsilon_t \right] = E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] \), so

\[
\frac{\partial^2 E_t [\ln(g_{t+1})] | \epsilon_t}{\partial \epsilon_t^2} = \left( E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] \right) \left( \left[ E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2 \right] - E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] \right) + 2E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] \\
- \frac{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2}{E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2}.
\]

Since debt may be zero, for expected utility to be concave in \( \epsilon_t \), it is necessary for this expression to be negative. Since \( \left( E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right]^2 \right) - E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] \) is second order in relation to the following term, this condition is always satisfied. This is specially easy to see for \( \xi \to \infty \), when \( E_{t+1} \left[ \frac{1}{\theta_{t+1}} \right] = \frac{1}{\theta + \epsilon_t} \), and likewise for other expectational terms, so

\[
\frac{\partial^2 E_t [\ln(g_{t+1})] | \epsilon_t}{\partial \epsilon_t^2} = -\frac{1}{(\theta + \epsilon_t)^2} \left( \left( \frac{1}{(\theta + \epsilon_t)^2} \right) - \frac{1}{(\theta + \epsilon_t)^2} \right) + 2\frac{1}{(\theta + \epsilon_t)^2} \frac{1}{(\theta + \epsilon_t)^2}
\]

\[= -\frac{2}{(\theta + \epsilon_t)^2} < 0.
\]

Concavity of the expected utility of public good in \( \epsilon_t \) implies that as past competence increases, the decreasing marginal utility of consumption is not offset by its marginal effect on the positive supply response of the public good to past competence.
References


