A THEORY OF KINKED DEMAND CURVES: DYNAMIC GAME THEORY AND PRICE RIGIDITY.

Schiaffino, Pablo

A Dixit Investment In Entry-Deterrence Approach.

PABLO SCHIAFFINO¹

(The University of Warwick)

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[Based on Dixit (1980) seminal paper, I show that the role of investment in entry-deterrence leads to a kinked demand curve faced by the leader. Several results emerge. First, I derive a kinked demand curve using modern theoretical formulations from game theory. Second, I derive the value of the kinks as a function of the parameters of the model. Third, I find a discontinuous kinked demand curve, where real price rigidity emerges. Four, I find that the degree of rigidity tends to increase as the market goes to monopoly. In the line of Maskin and Tirole (1988), this is the second paper which motivates the tripartita connexion: the kinked demand curve, real price rigidity and game theory. Others new findings are also discussed.]

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Back in 1939, Paul Sweezy introduced the concept of the kinked demand curve into academy generating a round of discussions that still continues nowadays. The Sweezy (1939) story is an oligopoly market structure where a price change by one firm, generates a not equal response by the others. In Sweezy’s story, when one firm increases its price, the others do not follow it. On the other hand, when one firm decreases its price, the others respond accordingly generating a price competition. The idea behind this argument is that price increase (ceteris paribus) motivates costumers to search for alternative suppliers, and from the perspective of the firm who does the change, this result in a more elastic demand. Sweezy argues that downward prices generate an immediate response (downward) by the rest: competitors do not follow price increase but do follow price decrease and hence, there are no incentives under these conditions to reduce prices. The resulting effect, due to prices variations, generates a kink at the market focal price2.

The kinked demand curve has a discontinuous marginal revenue at the kink. The above figure presents a kinked demand curve and the corresponding marginal revenue. If the marginal cost lays between the discontinuous gap, given by the distance between A and B, small changes to the marginal cost does not change the firm’s optimal price. Therefore, prices are rigid with respect to small changes in costs (recall that optimality conditions imply that marginal revenue equals marginal cost at equilibrium).

2 It is exactly the price where the firm does not want to change due to the response by the others.
The first seminal paper presented by Sweezy generated controversy, especially from the Chicago school who argued that prices adjust rapidly. However, Sweezy admits the shortcoming of the theory remarking that no explanation neither nor formalization are given to explain how the equilibrium price emerges.

Hall and Hitch (1939) presented a theoretical and empirical paper in order to sustain Sweeze’s idea. They first present evidence showing that rivals match price cuts but no price increase. Once again, the demand curve has a kink as Sweeze sustains. But more provocative is the explanation on how the focal price is reached. Totally contradictory respect to the classical concept of profit maximization, in Hall and Hitch story firms set prices based on historical prices and costs. While Sweeze left too many questions unanswered, Hall and Hitch presented an empirical observation with an unsatisfactory theoretical explanation.

The predictive power of the theoretical formulations was indeed very weak and the majority of the criticism came from the Chicago School, particularly from Stigler (1978). The kinked demand curve definitely not come from microfoundations theory- as Stigler holds- and contradicts the idea of market clearing equilibrium. But most important, the kinked demand may generate price stickiness which violates the formulation of efficient markets and fully adjustment prices. Stigler points out that the model is indeed incomplete and charges over Sweeze unanswered question: how is the focal price determined and how does the kink emerges?

The Chicago school together with the Lucas critique buried the idea of the kinked demand curve as its theoretical and empirical conjectures could not be taken seriously. It was over until the late seventies.

The kinked demand curved revival emerges during the eighties, but up to today literature about the subject is actually scarce. The most influential paper is due to Maskin and Tirole (1988) where two firms set a focal price equal to the monopoly one. Departing from here, if one of the firms raises its price, the rival does not follow it (hence, it continues charging the focal price). But if one of the firms cuts its price, the rival sets a lower one and war price starts (war of attrition). In the model, both firms have equal reaction function and there is an
asymmetry at the time when firms set prices. The outcome of the model is a Markov perfect equilibrium in which firms use trigger strategies; and the firm who makes the undercut has a short run gain which is lower than the total payoff of no deviating and setting the focal price. Hence, the important message here is that the kinked demand curve emerges from rational strategic behaviour. Despite the analytical and formal presentation of the paper, in this work the kinked demand curve could be well motivated under new theoretical formulations, particularly using tools from game theory.

Bashka (1988) continues the line of game-theoretical models to motivate a kinked demand curve. He shows a kinked demand curve with a focal price equal to the minimal common price outcome. But most importantly, he tests his model empirically using panel data surveys from manager's price setters and he concludes that they face a kinked demand curve. In a more recent paper, D.Sen (2004) uses a Stackelberg oligopoly to derive a kinked demand curve. Under some no heroic assumptions, he shows that the equilibrium price is rigid with respect to small cost changes.

The motivation in this paper is simple. As I described above, only two papers work out on this idea; Maskin and Tirole and D.Sen do provide theoretical formulations and the rest (even a few) are quite empirical: actually it is the absence of modern literature related to the kinked demand theory that motivates this paper.

In this paper I propose to refill the empty spaces with new theory in order to continue the line of the above mentioned works. Here, I study how a dynamic game generates a kinked demand curve and hence, real price rigidity may exist under some assumptions. Setting a theoretical framework in the type of Dixit (1980) where firms have to decide capacity and quantities, I first show how strategic behaviour leads to a kinked demand curve. The most important results behind this model are: 1) A new formulation of a kinked demand curve using strategic behaviour from a published paper (Dixit); 2) The exact values where the kinks emerge as a function of the parameters (such as cost of entry) of the model; 3) The degree of real price rigidity as a function of the number of firms in the market and possible implications for price rigidity literature; 4) The existence of investment in capacity as credible
commitment to defer other firms entry; and finally 5) I show that concavity of the kink does not depend on general demand properties (for example, to demystify the idea that concavity of demand is necessary to generate a concave kink; that is, I say it is false showing that a convex demand leads to a concave kink).

This paper continues as follows. Section 1 presents a theoretical model for a kinked demand curve. Section 2 is dedicated to discuss the results and the assumptions behind the model. Finally, I state some conclusions.

**A model for a kinked demand curve.**

The economy framework is due to Dixit (1980) and consists in a list of players, strategies and benefits on each eventual outcome:

Stage 1: Firm A chooses $k_A^*$ (capacity).

Stage 2: After having observed $k_A^*$, firm 1, 2 and 3 decide whether to enter the market. If firm 1, 2 and 3 enter the market, all three choose its capacity. I assume that this capacity can be adjusted downward or upward at no cost extra costs in stage 3 at the same time the four choose quantities.

Stage 3: Simultaneously, firm A, 1, 2 and 3 choose quantities and all four achieve an oligopoly Cournot-Nash equilibrium. Simultaneously too, firm A has the possibility to increase its capacity ($k_A \geq k_A^*$) at a certain cost which I specify below.

If none of the firms enter, firm A operates as a monopoly and output cannot exceed capacity. Each of the potential entrant firms, if they decide to enter, face a fixed cost of entry $(f_1, f_2, f_3)$ where $f_1 < f_2 < f_3$. The general expression for the function cost of firm $i$ for a given level of capacity is given by:

$$Total \ Cost_i = f_i + \alpha_i k_i + \beta_i q_i$$

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3 One possible justification of this assumption, compared to the one that holds for firm A, is what Tirole calls the “short-run” decision- see Tirole (1998). In the short run, a firm has the possibility to get a refund or buy extra capacity at no cost. In fact, under all assumptions here, I can reduce the model to two stages but it is simpler to expand it to three for illustrations purposes. This assumption avoids problems of “corner solutions” for firms 1, 2 and 3 at the sub-game of Stage 3.
Firm 1, 2 and 3 face the same unit cost per unit of installed capacity and the same unit cost per quantity produce (recall that the three face no cost of capacity adjustment). I can write the total cost function for firm 1, 2 and 3 in the form of:

\[ \text{Total Cost}_i = f_i + \beta q_i + \alpha k_i \]

On the other hand, firm A faces no cost of entry \((f_A = 0)\). Investment in capacity has a unitary cost of \(\alpha_A\) and output production has a unitary cost of \(\beta_A\). In the third period, if the firm wants to increase its capacity, it faces a unit cost of \(\alpha'_A\) where \(\alpha'_A > \alpha_A\).

I assume a linear demand function \((p = a - q)\) which is useful for tractability purposes. In the next section I discuss extensively the implication of demand functions. The result is that concavity (or linearity) of demand function is not necessary to get a concave kink.

**Stage 3.** (All four choose quantities; firm A might increase capacity at certain cost; firm 1, 2 and 3 may increase/decrease capacity at no cost).

Consider first that firm A has installed a fixed level of capacity equal to \(k^*_A\). Let’s define \(\tilde{q}_A\) as the optimal quantity produced by firm A. In the third stage, if it decides to produce a certain quantity such as \(\tilde{q}_A \leq k^*_A\) the total cost are given by \(\alpha_A k^*_A \beta_A \tilde{q}_A \). Because firm A does not exceed the installed capacity, the marginal cost in terms of \(q_A\) is equal to \(\beta_A\). On the other hand, if firm A wishes to go beyond the installed capacity, it must invest in extra capacity; hence, if \(\tilde{q}_A > k^*_A\), the total costs in this case are \(\alpha_A k^*_A + \alpha'(A) (\tilde{q}_A - k^*_A) + \beta_A \tilde{q}_A\) and the marginal cost is now equal to \(\alpha'_A + \beta_A\).

Using the above results and taken as given the quantity produced by firm 1, 2 and 3, I derive the reaction function of firm A. Because I have shown that the marginal cost depends on outputs below/above \(k^*_A\), figure 1 shows the two reaction functions of firm A for a fixed cohort level of \(k^*_A\).
From the perspective of firm A, up to $k_A^*$, the marginal cost is only $\beta_A$ and the reaction function is:

$$R_A(q_1, q_2, q_3) = \frac{a - \sum_i^3 q_i - \beta_A}{4}$$

Beyond $k_A^*$, the marginal cost now turns to $\alpha_A' + \beta_A$ and the reaction function is:

$$R_A'(q_1, q_2, q_3) = \frac{a - \sum_i^3 q_i - (\alpha_A' + \beta_A)}{4}$$

As Dixit states, point $X$ is the optimum quantity set by firm A when expansion cost matters, and $Y$ is the optimum quantity set when there is sufficient capacity installed. These two hold when the level of output by each of the other firms is set to zero.

Similarly, the reaction functions for firm 1, 2 and 3 can be written as:

$$R_1 = R_2 = R_3 = \frac{a - q_A - \beta}{4}.$$ 

The reaction function of firm A for a given level of $k_A^*$ and the reaction function of firm 1, 2 and 3 (denoted by $R' R$) are illustrated in figure 2.
There is an oligopoly Nash equilibrium at the intersection of the two reaction curves. Let’s explain this in detail. The $E1$ intersection \( \left( \frac{1}{13}a - \frac{4}{13}\alpha' + \frac{3}{13}\beta - \frac{4}{13}\beta_A; \frac{3}{13}a + \frac{1}{13}\alpha' - \frac{4}{13}\beta + \frac{1}{13}\beta_A \right) \) gives the Nash equilibrium when firm A goes beyond its current capacity; while $E2$ \( \left( \frac{1}{13}a + \frac{3}{13}\beta - \frac{4}{13}\beta_A; \frac{3}{13}a - \frac{4}{13}\beta + \frac{1}{13}\beta_A \right) \) gives the equilibrium when firm A does not go beyond. Because firm A has the advantage in choosing the capacity level first, it can determine which reaction function will prevail in the post-entry oligopoly. To see this, from figure 1 and 2 it is possible to observe that for a capacity choice such as \( k_A^* \leq \frac{1}{13}a - \frac{4}{13}\alpha' + \frac{3}{13}\beta - \frac{4}{13}\beta_A \), the third period sub-game perfect Nash equilibrium holds at $E1$. On the other hand, for a choice of \( k_A^* \geq \frac{1}{13}a + \frac{3}{13}\beta - \frac{4}{13}\beta_A \) the sub-game perfect Nash equilibrium holds at $E2$. The intermediate case, when capacity is \( \frac{1}{13}a - \frac{4}{13}\alpha' + \frac{3}{13}\beta - \frac{4}{13}\beta_A \leq k_A^* \leq \frac{1}{13}a + \frac{3}{13}\beta - \frac{4}{13}\beta_A \), the sub-game perfect Nash equilibrium is at any point of the heavy line denoted between $E1$ and $E2$ and here $q_A = k_A^*$ (see the appendix for more illustrations).

Notice that capacity levels above \( \left( \frac{1}{13}a + \frac{3}{13}\beta - \frac{4}{13}\beta_A \right) \) are not credible threats. Consider firm A investing in capacity above this value. Because the Nash equilibrium holds at $E2$, it would not be optimal for firm A to carry a costly empty threat. Firm 1, 2 and 3 know with certainty that $E2$ is the equilibrium for any value of $k_A^*$ such as \( k_A^* \geq \frac{1}{13}a + \frac{3}{13}\beta - \frac{4}{13}\beta_A \). Once again, firm A would carry a costly and empty threat. Conversely, firm A would not set a capacity less than
\( \left( \frac{1}{13} a - \frac{4}{13} \alpha_A' + \frac{3}{13} \beta - \frac{4}{13} \beta_A \right) \). If entry occurs, it wants more capacity while if entry is not to occur it wants a capacity of at least \( X > \frac{1}{13} a - \frac{4}{13} \alpha_A' + \frac{3}{13} \beta - \frac{4}{13} \beta_A \).

Recall that firm 1, 2 and 3 choose capacity at stage 2 and at the beginning of stage 3 it is fixed; thus a case where optimum production at stage 3 is beyond (or below) the installed capacity is possible. However, if this were the case, at this stage the three firms would increase (reduced) capacity immediately at no extra cost in order to equal it to the optimum production.

So far, I have shown that the quantity produced in equilibrium by any of the firms lies between \( E1 \) and \( E2 \). Whether or not the quantity produced by firm A equals the investment in capacity remains to be seen in the equilibrium path (even though I have provided some intuition in the lines above). Furthermore, I have not say anything yet about the *ex ante* decision whether to enter from the perspective of firm 1, 2 and 3.

It is worthwhile to mention that the post-entry game leads to a Nash equilibrium, where firm A has the advantage to choose the initial level of capacity (stage one) and then to increase it or not (third stage) at certain cost. This gives firm A the power to manipulate the initial conditions of the game and firm 1, 2 and 3 will enter the market if the profits are positive. Aware of this, firm A chooses the level of capacity that maximises its profit and this may result in a level that prevents or allows the rest of the firms to enter.

**Stage 2.** (Firm 1, 2 and 3 choose capacity).

Here, firm 1, 2 and 3 observe the level of capacity set by firm A and decide whether to enter. Enter to the market implies that each of the firms have to choose its own capacity level. Introducing the values obtained in stage 3 for \( E1, E2 \) and the intermediates between them into the profits functions gives:

\[
\begin{align*}
\pi_1 &= (a - (q_A + q_1 + q_2 + q_3))q_1 - \beta q_1 - \alpha k_1 - f_1 \\
\pi_2 &= (a - (q_A + q_1 + q_2 + q_3))q_2 - \beta q_2 - \alpha k_2 - f_2 \\
\pi_3 &= (a - (q_A + q_1 + q_2 + q_3))q_3 - \beta q_3 - \alpha k_3 - f_3
\end{align*}
\]
Each of the firms has to choose a capacity level that maximizes its profits. Notice that 
\((q_A, q_1, q_2, q_3)\) are in function of exogenous parameters that comes from stage 3. From here, 
it is immediate that the three firms have no commitment in capacity and this implies that for 
all levels of output, they will match the production level to the capacity despite that they can 
adjust it at no extra cost in the third stage. Notice that there is no point in accumulating 
capacity that is not used for production and setting a capacity level below the optimum output 
would be, of course, not optimal. Hence, firm \(i\) (for \(i=1,2,3\)) produces at full capacity level, 
that is \(q_i = k_i\).

**Stage 1.** (Firm A chooses capacity).

At this stage, firm A knows that the rest are going to set capacity equal to production. In 
addition, it knows the equilibrium values from stage 3. Hence, using this information, firm A 
would choose a capacity level that maximizes its total payoff. I can write firm’s A profits 
function as:

\[
\max_{k_A} \pi = \begin{cases} 
1 & (a - q_A(E1) - \sum_i^3 q_i(E1))q_A(E1) - (a_A k_A + \beta_A q_A(E1) + a'_A(q_A(E1) - k_A)) \text{ subject to } k_A < q_A(E1) \\
2 & (a - q_A(E2) - \sum_i^3 q_i(E2))q_A(E2) - (a_A k_A + \beta_A q_A(E2)) \text{ subject to } k_A \geq q_A(E2) \\
3 & (a - q_A - \sum_i^3 q_i)q_A - (a_A k_A + \beta_A q_A) \text{ subject to } q_A(E1) \leq k_A \leq q_A(E2)
\end{cases}
\]

As in stage 2, \((q_A, q_1, q_2, q_3)\) are in function of exogenous parameters; hence fixed. Consider 
[3]. From the equilibrium in the third stage, we know that for this range of \(k_A\), output always 
equals capacity. Consider [2]. The optimal level of investment here is exactly where \(q_A = k_A\).

In this case increasing capacity above \(q_A(E2)\) would generate only a decrease in profits.

Finally consider [1]. The profit function is increasing in the value of capacity investment; 
when \(k_A\) is close to \(q_A(E1)\), that is when \(\lim_{k_A \to q_A} \pi(k_A)\), the profit function approximates to 
a maximum. This maximum, as in the previous cases, is when \(q_A\) is close to \(k_A\).

From the three possible cases, the optimal choice of capacity is the one that equals capacity 
to output. Or in other words, firm A would choose a level of investment that equals the 
quantity produced in the third period. This implies that, in the third stage, firm A will not 
increase the capacity and this is the best strategy.
Now it is possible to analyze the *ex ante* decision about whether to enter from the perspective of firm 1, 2 and 3. Sep-up costs are now important and have to be taken into account. I have shown that the post-entry equilibrium lies between $E1$ and $E2$ (with entry or no entry) and firm A would be producing an output equal to the capacity selected at stage 1.

Recall that for every firm, output is equal to capacity, hence:

\[
\begin{align*}
\pi_1 &= (a - \alpha - \beta - (q_A + q_1 + q_2 + q_3))q_1 - f_1 \\
\pi_2 &= (a - \alpha - \beta - (q_A + q_1 + q_2 + q_3))q_2 - f_2 \\
\pi_3 &= (a - \alpha - \beta - (q_A + q_1 + q_2 + q_3))q_3 - f_3
\end{align*}
\]

Let’s write $c = \alpha - \beta$. Assume that $\pi_i(E1) > 0$ and $\pi_i(E2) < 0$ for every $i=1,2,3$. The profits for each of the firms decrease monotonically as we move from $E1$ to $E2$. This implies that there exists a point $Z$ between $E1$ and $E2$ where the profits of the firm $i$ are zero. Because the costs of entry are asymmetrical, this $Z$ point would be different for firm 1, 2 and 3. Using again the reaction functions from profit maximization by firm 1, 2 and 3:

\[
R_1 = R_2 = R_3 = \frac{a - c - q_A}{4}
\]

I am looking for the capacity level (set by firm A) that prevents entry for each one of the firms.

Replacing the above expression in the corresponding profit functions, I get:

\[
\begin{align*}
\pi_1 &= \left(\frac{a - c - q_A}{4}\right)^2 - f_1 \\
\pi_2 &= \left(\frac{a - c - q_A}{4}\right)^2 - f_2 \\
\pi_3 &= \left(\frac{a - c - q_A}{4}\right)^2 - f_3
\end{align*}
\]

Firm A has the power to commit to high levels of quantity (and as we are going to see later on, this implies high levels of capacity) and hence, to deterrence entry. Due to the asymmetry in the cost of entry, for a given value of $q_A$, firm 3 is the first who gets out of the

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*If $\pi_i(E1) < 0$ entry is irrelevant. Neither of the firms would enter because profits in the post-entry equilibrium are always negative. If $\pi_i(E2) > 0$, all possible profits in the post-entry equilibrium are positive. There is no way that firm A can deter entry here. These two cases are ruled out from this paper, but it is worthwhile to mention it.*
market. Too see this notice that $\pi_3 = 0$ if $q_A = a - c - 4\sqrt{f_3}$. Consider now a number of cases:

Case 1: $q_A(E1) \leq q_A \leq a - c - 4\sqrt{f_3}$

In this case, all firms enter and the four firms compete in quantities. The residual demand faced by firm A is given by $D_A^{res} = D(q_A) - q_1(q_A) - q_2(q_A) - q_3(q_A)$. Replacing with the reaction curves I get $D_A^{res1} = \frac{a+3c}{4} - \frac{1}{4}q_A$. The corresponding marginal revenue is giving by $MR_A^{res1} = \frac{a+3c}{4} - \frac{1}{2}q_A$. 

Case 2: $a - c - 4\sqrt{f_3} \leq q_A \leq a - c - 3\sqrt{f_2}$

For a capacity level between $a - c - 4\sqrt{f_3}$ and $a - c - 3\sqrt{f_2}$, firm 3 stays out of the market and the maximization problem by firms 1 and 2 implies:

$q_1 = q_2 = \frac{a - c - q_A}{3}$

In this case, the residual demand is given by $D_A^{res2} = \frac{a+2c}{3} - \frac{1}{3}q_A$ and the marginal revenue is given by $MR_A^{res2} = \frac{a+2c}{3} - \frac{2}{3}q_A$.

Case 3: $a - c - 3\sqrt{f_2} \leq q_A \leq a - c - 2\sqrt{f_1}$

Finally, only firm 1 stays in the market and it is a duopoly. Now, the reaction function is simple:

$q_1 = \frac{a - c - q_A}{2}$

In this case, I find the residual demand given by $D_A^{res3} = \frac{a+c}{2} - \frac{1}{2}q_A$ and $MR_A^{res3} = \frac{a+c}{2} - q_A$.

Case 4: $q_A(E2) \geq k_A \geq a - c - 2\sqrt{f_1}$

For this specific cohort value of capacity, there is no entry and only firm A gets the entire market facing the entire demand. Hence, $D_A^{res4} = a - q_A$ and $MR_A^{res4} = a - 2q_A$.

Figure 3 presents graphically the four cases together. At each of the kinks the residual demand is discontinuous. Two effects emerge and have to be mention. Consider the first kink. Firstly, if the firm A sets that quantity, firm 3 is out of the market. At the left of the kink number one, the residual demand is the one where four firms operate in the industry. At the
right, the residual demand is the one with three firms. Because now one of the firms is out of
the market and the total output is lower, at kink number one, the residual demand with three
firms has a higher price compare with the residual demand with four firms. Secondly, the
slope of residual demand function becomes more negative as the industry tends to
monopoly. The combination of this two effects in a situation where the second is stronger
than the first, leads to the following statement at kink number one:

\[ MR_A^{res \left( \text{kinked 1} \right)} \geq MR_A^{res \left( \text{kinked 1} \right)} \]

As long as I assume that \( f_3 \leq \left( \frac{a-c}{8} \right)^2 \) the above is guarantee. This also guarantees that:

\[ MR_A^{res \left( \text{kinked 2} \right)} \geq MR_A^{res \left( \text{kinked 2} \right)} \]
\[ MR_A^{res \left( \text{kinked 3} \right)} \geq MR_A^{res \left( \text{kinked 3} \right)} \]

The discontinuity of the residual demand includes a new result compared to the one
presented by Sweeze, where the residual demand was continuous. Hence, under a Dixit
theoretical framework with quantity competition and capacity setting, there exists a
discontinuous kinked demand curve.

![Figure 3](image-url)
Discussion and results.

*The kink is a function of the costs of entry and the irreversibility of installed capacity makes investment a credible threat.* A new feature compare to the original kinked demand curve is that the value of the kink (the capacity set by firm A that puts one of the firms out the market) is a function of the entry cost. The correlation between the capacity set by firm A and the cost of entry is negative, suggesting that a lower cost of entry for the rest implies a higher level of capacity by firm A. Because investment in capacity today has strategic effect over the potential entrant firms, a higher level of investment by the leader reduces the equilibrium price, affecting the profitability of the firms and setting a potential barrier to entry; or in other words, a higher level of capacity reduces the marginal profit from investing for the other three firms. Compare with the case of simultaneous equilibrium (where firms set its capacity level at the same moment), Tirole (1988) shows that capital accumulation by the leader firm is higher. This suggests that firm A may overinvest in order to deter others entry, when the firm is the leader. By this reason, it is said that capacity accumulation has a strategic effect and is the leading piece behind the kinked demand curve.

*Real price rigidity.* One of the most important features about the kinked demand curve is the existence of real price rigidities. Too see this notice that if the marginal cost intersects the marginal revenue at one of the kinks (the exact point where the marginal revenue is discontinuous), small changes in the value of $\alpha_i$ (assume $\beta_i$ does not change at all) does not change the equilibrium price, and hence prices are rigid with respect to small changes in the cost function. I present this result here and the foundations are not very different from Sweezey. For the sake of transparency, notice that this paper does indeed motivate the kinked demand curve story with formulations of game theory: I do explain kinks, and the kinks are in function of the parameters of the model. However, price rigidities hold under some particular configuration of the parameters (for example, the marginal cost has to intersect the marginal revenue at one of the kinks) and I do not provide any theoretical framework of why the economy may reach Sweezey’s focal price. Up to the moment, the
only paper which provides a story about this issue is Tirole and Maskin (1988) and this is a

clear difference between the two.

But the interesting feature here is the measure of the degree of rigidity which is given by the

longitude of the discontinuity at the kink.

For example, at the first kink of the model this measure is given by: \( MR_A^{res1}(a - c - 4\sqrt{f_3}) - MR_A^{res2}(a - c - 4\sqrt{f_3}) \). In terms of one dimensional Euclidean distance and simplify (recall that \( MR_A^{res1}(\text{kinked } 1) \geq MR_A^{res2}(\text{kinked } 1) \)):

\[
\varphi(kink_1) = \left( \frac{a + 3c}{4} - \frac{1}{2}(a - c - 4\sqrt{f_3}) \right) - \left( \frac{a + 2c}{3} - \frac{2}{3}(a - c - 4\sqrt{f_3}) \right)
\]

\[
\varphi(kink_2) = \left( \frac{a + 2c}{3} - \frac{2}{3}(a - c - 3\sqrt{f_2}) \right) - \left( \frac{a + c}{2} - (a - c - 3\sqrt{f_2}) \right)
\]

\[
\varphi(kink_3) = \left( \frac{a + c}{2} - (a - c - 2\sqrt{f_1}) \right) - \left( a - (a - c - 2\sqrt{f_1}) \right)
\]

Given a pair of \((a, c)\) it always holds that \( \varphi(kink_3) > \varphi(kink_2) > \varphi(kink_1) \) when \( \left( \frac{a - c}{8} \right)^2 \geq f_3 > f_2 > f_1 \geq 0 \). The idea behind Figure 4 is that \( \varphi(kink_i) \) for \( i = 1, 2, 3 \) is the longitude at each of the kinks as a function of the entry costs (and other parameters). Notice that the distance tends to increase as we move from the first kink to the third suggesting the degree of rigidity increases as the numbers of firms decrease. The asymmetry in the cost of entry, of course, plays a crucial role.
To see this, consider two equal markets with the same structure. Assume that in the first market, the equilibrium price holds at the first kink while in the second market holds at the second (this may occur if the two economies are exactly the same except in the marginal cost). When a shock to function cost occurs, there is a big chance that real price rigidity remains in the second economy than in the first. Once again, the longitude at the discontinuous point of the marginal revenue tends to increase as the market structure goes to monopoly. This implies that the margin for real price rigidity decreases as the number of firms in the economy becomes bigger. Hence, the market structure is an important aspect to take into account when price rigidity is study under imperfect competition.

Concavity of demand: The resulting residual demand I derive here has a kink that points outwards. I call this a concave kink. However, the nature of this concavity does not depend of the demand curve properties. This subsection is motivated due to following seminar question: Can you proof that concavity of demand function is not necessary to get a concave kink? In the next lines, I show that a convex demand curve gives a concave too, but first some comments about concave/convexity should be mention.

A convex kink also generates real price rigidity. The only difference with what I had done here is the direction of the kink. Elmore et al.(2006) explains this idea explicitly: Sweezy also presents an alternative model for a kinked demand curve in which the kink points in the
opposite direction. This alternative model assumes that rivals do not match decreases in price, but match increases. Sweezy explains that this situation might occur if firms grant “secret” discounts to preferred customers and rival firms are not aware of the price decreases. In addition, Sweezy assumes that rival firms will match rises in price, which occurs when there are price leaders in an industry. As in the previous model, a change in the marginal cost does not imply a change in the price, directly contradicting the Chicago’s economist’s assumption that prices adjust rapidly.

To see that concavity of the kink does not depend of demand curvature, consider the following simplify version of the previous model. There are two firms in the market, firm 1 and 2, and they compete in quantities (no capacity selection). Firm 2 observes firm’s 1 quantity and decide whether or not to enter (there is no fixed cost). Both face a marginal cost equal to\( c \). The demand function has the following form:

\[
p = (a - Q)^{\frac{1}{w-1}}
\]

\[
Q = q_1 + q_2
\]

This type of demand function is due to Data and Dixon (2000). When \( w = 2 \), it is a classical linear demand curve. If \( w < 2 \), the demand is strictly convex; and if \( w > 2 \), the demand is strictly concave. The profit maximization by firm 2 implies marginal revenue equals marginal cost, hence:

\[
\frac{-q_2}{w - 1} (a - Q)^{\frac{2-w}{w-1}} + (a - Q)^{\frac{1}{w-1}} = c
\]

Solving for \( q_1 (a, c, w, q_2) \) and replacing it over \( p (a, w, q_1, q_2) \) I get an expression for the residual demand curve. Due to complexity of the algebraic results, I approach the kinked demand curve face by firm 1 graphically in figure 5:
For example, consider the arbitrary case when $w = \frac{3}{2}, a = 100$ and $c = 30$. The demand is now convex. The profit maximization problem by firm 2 implies the reaction function given by

$$q_2 = \frac{200}{3} - \frac{1}{3} \sqrt{q_1^2 - 200q_1 + 10090} - \frac{2}{3} q_1.$$  
Replacing this expression into the demand function in order to get the residual demand curve:

$$D_{1 res} = 100 - q_1 - \left( \frac{200}{3} - \frac{1}{3} \sqrt{q_1^2 - 200q_1 + 10090} - \frac{2}{3} q_1 \right)^2$$

$$= \left( \frac{1}{3} \sqrt{q_1^2 - 200q_1 + 10090} - \frac{1}{3} q_1 + \frac{100}{3} \right)^2$$

for $q_1^* \leq 100 - \sqrt{30}$

For any value beyond $q_1^*$ the demand is the monopoly one, that is $(100 - q_1)^2$. Figure 6 illustrates this situation amplify at the kink. Despite the convexity of the total and residual demand, the kink as before, is concave.
Conclusions.

Sweezey gave birth to the story of the kinked demand curve and later on, Chicago school buried it. During the eighties, the idea was back on track using modern formulations of game theory. The common view in that time was to connect the kinked demand curve, strategic interactions, and price rigidity. The novelty of course, was the strategic interaction factor. Despite the fact that some empirical and theoretical literature emerged, the actual research over the kinked demand curve is vague. In this paper, I contribute with a model based on Dixit (1980) seminal paper where the role of investment in entry-deterrence leads to a kinked demand curve faced by the leader. From here, a number of results emerge. First, I depart from the idea that investment in capacity is a credible commitment to defer other firm’s entry. Second, I derive the value of the kinks as a function of the parameters of the model (particularly interesting, the costs of entry). In the Sweezy’s original kinked demand curve, there was no specification and no values for the kinks. Third, I derive a discontinuous kinked demand curve where price rigidity prevails. This result provides new fundamentals for real price rigidity literature as it is indirectly the formulation of dynamic games and price rigidity that is established. Finally, I find that the degree of rigidity tends to increase as the market tends to monopoly. This result was not found in previous literature and suggests that the structure of an oligopoly market may play a crucial result over nominal shocks and real
effects for a given economy. Others results, such as implications of convex demand function over kinks also were studied.

References.


Appendix 1.

\[ q_A \geq k_A^* \]

\[ q_A = k_A^* \]

\[ q_A \leq k_A^* \]