OVERINVESTMENT PATTERN IN THREE-STAGE ENTRY DETERRENCE MODELS

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Abstract

The present paper deals with an entry deterrence model in which each firm decides about the level of a strategic investment variable that increases marginal benefits of its own tactical variable (quantities) and one firm (the incumbent) moves first. In the last period, both firms compete at the market. Emphasizing that the overinvestment pattern found in Fudenberg and Tirole’s (1984) model is benchmark-specific, our main goal is establishing baselines where this pattern is still present according to a Ware’s (1984) three-stage framework. We demonstrate that this is true in case we confront the results against either an strategic baseline or the solution obtained by a regulator whose objective is to maximize aggregate benefits in a second-best framework with voluntary participations of firms and where it can only choose the level of strategic investment variables of both firms but competition in quantities remains. The latter benchmark has direct policy implications for exporting and infant industries and, remarkably, it does not require an assumption about the sign of strategic investments best-response’s slope.

Keywords: Entry deterrence, Fudenberg and Tirole’s taxonomy, supermodular game.

1 Introduction

Over about the last thirty years, Industrial Organization field has been concerned, among other topics, with firms’ strategic behavior to gain market power. Following

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Wilson (1990), the entry deterrence’s models which have arisen can be split into three broad categories, namely Preemption, Signaling and Predation. In particular, preemption models apply Schelling’s idea of Strategic Behavior and are characterized in a general and a schematic way by Fudenberg and Tirole (1984) and Bulow, Geanakoplos and Klemperer (1985). They all have a similar structure. At a first stage, there is only one firm (the incumbent) which has the option to make a decision that entails a commitment. This can take the form of a strategic variable that affects either its own revenue or costs and which, by its sunkness nature, makes the decision irreversible. This is carried out with the aim of getting an advantage over a potential entrant which, in case it decides to enter, would compete in the market with it. The strategic investment is valuable for the incumbent by its level is seen by the potential entrant before the entry decision and competition in market takes place. Finally, at a second stage, there is competition in market by quantities or prices. In particular, Fudenberg and Tirole (1984) establish a taxonomy of strategies for the incumbent’s choice. They show that just by knowing the nature of the strategic variable (this means, whether it increases or decreases marginal benefits of tactical variables) and the tactical variable best-response function’s slope (which defines strategic substitutes or complements), it is possible to characterize qualitatively incumbent’s strategy. The characterization involves comparing the feasible solutions against a specific benchmark and ascribing a pattern of under or overinvestment to each strategy. Despite these papers dating from the ’80s, preemption models still prevail as a standard framework to study entry deterrence issues.

In the present paper, we address the following issues. In the first place, we depart from Fudenberg and Tirole’s (1984) setup in two aspects. We follow Ware’s (1984) treatment of the entrant’s strategic variable nature and a three-stage model arises where the entrant is able to influence the postentry output game although the incumbent maintains its position by making its strategic investment decision before the entrant does. In addition, we modify the market stage game in order to convert it into a supermodular game such that multiple equilibria at market stage are allowed. This implies that as we do not make any assumption about concavity or convexity of either demand or cost function, it allows us to cover a broader category of market configurations.

In the second place, we deal with the fact that the taxonomy put forward by Fudenberg and Tirole (1984) has become so widespread that it has lead to confusion such that the term overinvestment is immediately associated to the case where investment variable increases firm’s own marginal benefits (investment makes firms "tough" in Fudenberg and Tirole’s terminology) and competition in market place is by quantities (more generally, strategic substitutes). Nonetheless, this pattern is not longer true in case we confront the solution with alternative benchmarks. Furthermore, the implications of a model where overinvestment exists is benchmark-specific as the results are derived from a comparison against a given one. Therefore, we address the issue of setting baselines where the overinvestment pattern is still true and characterize the result according to the baseline used as comparison.

The first baseline to take into consideration is the one used by Fudenberg and Tirole (1984), namely a strategic benchmark given by the open loop equilibrium and
monopoly for accommodation and entry deterrence, respectively. In comparison to a two-stage model and in order to obtain the overinvestment result, we need to make the additional assumption that strategic investments are substitutes. Moreover, as we have a supermodular market stage game, we find that the overinvestment result holds even for cases where multiple equilibria exist. This strategy allows the incumbent to have a first-mover advantage although we found that it is diminished in comparison to the two-stage model.

The comparison with a strategic baseline is interesting as it let us know that a value of incumbency exists and entails an overinvestment strategy in order to appropriate it. Notwithstanding, the results for this benchmark have a narrow political scope\(^1\). Therefore, we inquire about alternative benchmarks where the incumbent’s overinvestment result translates into a situation that calls for a reduction of investment in order to achieve a given policy maker’s objective. We show that this is the case when the benchmark is the solution a regulator would set in case its objective is to maximize aggregate benefits (with voluntary participation of firms) and is only capable of selecting the level of the strategic investment variable of each firm. Remarkably, unlike open loop baseline, the overinvestment pattern is robust to whether strategic investments are substitutes or complements. Maximizing aggregate benefits could be regulator’s aim if we were dealing either with firms which almost exclusively sell its product abroad (so that consumer’s welfare is not relevant) or with an infant industry with non-negligible entering cost that needs to capture extraordinary benefits in order to be developed. The same could occur if the industry is important for geopolitical reasons. What is more, we set the results in a context where the regulator cannot influence competition at the market stage. This turns to be a relevant case in international trade as countries adhered to World Trade Organization have a commitment not to subsidize exports or, at least, to gradually reduce them.

The rest of the paper is organized as follows. Section 2 establishes a background of the model and justify some considerations about it. Section 3 establishes the setup of the model. Section 3 characterizes both strategic investment and quantities decisions. Section for 4 does the same regarding firm’s behavior and, in addition, the solution of the model is provided. Then, in Section 5 and 6 we procede to make the comparison against strategic and aggregate benefits benchmark, respectively. Finally, in Section 7, we present some concluding remarks.

\(^1\) However, a potential policy application can be provided in a Brander and Spencer’s (1983) way. Let’s assume one of the firms is domestic and the other one from abroad, in addition to the domestic government being the only one able to establish subsidies to domestic strategic investment variable. In that case, if both firms makes strategic investment’s decision simultaneously, domestic government could give a subsidy with the objective of maximizing domestic firm’s benefits. As a consequence, domestic firm would have incentives to increase the level of its strategic investment variable, giving rise to the same solution that if it were an incumbent.
2 Background

Entry deterrence models lead us to make a reference about commitment or, more general, Schelling’s idea of strategic move. Schelling defines a strategic move\(^2\) as “one that influences the other person’s choice, in a manner favorable to one’s self, by affecting the other person’s expectation on how one’s self will behave\(^3\). As a result, the strategic move has to either change the set of choices available to the player or its payoffs. Furthermore, a necessary condition to be credible is that it is irreversible. If it is observed by players, the move not only changes own player’s incentives but also modifies opponent’s beliefs and, as a corollary, its optimal decision path from that point on.

Applying the concept to firms behavior, they take actions that changes the way competition in market is carried out. Evaluating these actions as strategic moves requires to identify the circumstances under which these properties are met. This lead us to consider two aspects: strategic move’s communication structure and its way to ensure commitment. Entry deterrence models tend to assume that an exogenous way to communicate the move exists.\(^4\) Instead, they tend to focus on the nature of the instruments used as a strategic move and its plausibility of being considered as irreversible. We can establish Spence (1977) and Dixit (1980) as the first models which treat endogenous barriers to entry within a game theory framework. Both use installation an investment (capacity) as a strategic move to alter a subsequent quantity competition stage.\(^5\) After these works, there has been a plethora of papers which considered different kind of strategic moves.\(^6\) Ware (1984) takes Dixit’s model up and makes a caveat: if both incumbent and entrant must incur in an investment that involves a sunk expenditure, a two-stage model would not reflect the situation in a proper way for many such models. Rather, a three-stage model would arise where entrant’s investment decision is made before market competition takes place. In addition to being a fairly natural modeling, it allows us to separate effects which a two stage model confounds. On the one hand, it exists a value of incumbency as incumbent moves first and, on the other hand, it exists an advantage for incumbent firm because entrant must make tactical variable’s decision incurring both variable and sunk costs.

Ware’s model is not general as is presented for the case of specific functional forms.\(^7\) In the present paper, we are concerned to a strategic variable that has an impact in pro-

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\(^2\)It is important to note that, as it is standard in the literature, we focus only on indirect strategic moves. This implies that the behavior of the opponents is modified only as a consequence of a change in the incentives of the player which has made the move. In other terms, in the context of this paper, a strategic move will not be concerned with those cases in which a player change the incentives to choose an action of other player than itself.

\(^3\)Schelling (1960), op. cit.

\(^4\)Besides, this is common knowledge between firms.

\(^5\)The main difference between both models lies in the fact that Dixit uses a subgame perfect equilibrium as solution concept.

\(^6\)See, for instance, Tirole (1990) or Shapiro (1988) for an exposure of some of these models.

\(^7\)Specifically, it is assumed a linear inverse demand function and constant variable costs up to full capacity.
duction technology through reducing monotonically marginal costs. At the same time there is a trade-off as investing in this variable is costly. In addition, as it is standard in the literature, we assume that an exogenous way to turn observable this move exists and is effective. It is important to note that, although we use a specific strategic move for the sake of exposure simplicity, the results we find apply to any strategic variable that increments the marginal benefits of tactical variable. This means that, instead of investment in research and development that reduces marginal cost, we could have taken another strategic variable that affects own firm’s revenue.

3 Setup of the model

Let \( q_i \) denote firm \( i \)'s quantities of a homogenous product which belongs to the set \( Q_i = \{0, \bar{Q} \} \). Let \( k_i \) denote the strategic investment variable of the \( i \)-th firm defined over the set \( K_i = \{0, \bar{K}_i \} \). Let subscript indexes 1 and 2 for variables refer to incumbent and entrant, respectively. The timing of the model is given by a three-stage model where first the incumbent chooses \( k_1 \) which entails sinking costs. Next, after seeing the level of \( k_1 \), the entrant makes a choice about whether enter or not. If it decides not to enter, the incumbent chooses a level of \( q_1 \) as a monopolist. If the entrant enters, it decides the level of \( k_2 \) which also involves sinking costs. After the level of \( k_2 \) being seen by the incumbent, incumbent and entrant compete in the market choosing simultaneously \( q_1 \) and \( q_2 \), respectively.

A strategy for the incumbent firm is a pair \((k_1, q_1(k_1, k_2))\) while for the entrant consists of \((k_2(k_1), 1(\Pi^2 - F \geq 0), q_2(k_1, k_2))\) where \(1(\cdot)\) is an indicator function which takes the value 1 if entrant enters and 0 if it does not. Defining, \(q^* \triangleq \{q_1(k_1, k_2), 1(\cdot), q_2(k_1, k_2)\} \) and \(k^* \triangleq \{k_1, k_2(k_1)\} \) a strategy profile is given by \((q^*, 1(\cdot), k^*)\). We use subgame perfection as equilibrium criterion.

In order to avoid tedious notation, throughout the text we use superscripts indexes 1 and 2 in functions to refer to incumbent and entrant, respectively. Moreover, for a differentiable function the subscripts refer to the variable we are deriving with respect to.

Let \( C^i : Q_i \times K_i \to \mathbb{R} \) denote the \( i \)-th firm’s cost function. It is assumed to be equal for all firms with \( C^i(q_i, k_i) \in C^2 \) on its domain and monotone increasing in \( q_i \) so that \( C^i(q_i, k_i) > 0 \) \( \forall q_i, k_i \). Furthermore, we assume that \( \lim_{k_i \to 0} C^i_{k_i}(q_i, k_i) < 0 \) \( \forall q_i \) and \( \lim_{k_i \to \bar{K}_i} C^i_{k_i}(q_i, k_i) > 0 \) \( \forall q_i \). Besides, by Young’s theorem, we have that \( C^i_{k_i}(q_i, k_i) = C^i_{k_kf_1}(q_i, k_i) \) \( \forall q_i, k_i \) and we establish that \( C^i_{k_i}(q_i, k_i) < 0 \) \( \forall q_i, k_i \). Furthermore, we set that \( C^i_{k_i}(q_i, k_i) > 0 \) \( \forall q_i, k_i \). We do not state an initial assumption about \( C^i_{k_i}(q_i, k_i) \) so that there could be a range of values where marginal costs are decreasing.

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8 For instance, we could have considered advertising, or more general, sales effort.
9 Actually, it is enough to define that \( Q_i = \mathbb{R}^+ \) as later we define \( \bar{Q} \) as the infimum quantity such that the demand is satiated. Then all strategies that \( q_i > \bar{Q} \) would be strictly dominated by \( q_i = 0 \).
Let $R^i : Q_i \times Q_j \to \mathbb{R}$ be the revenue function with $R^i (q_i, q_j) \triangleq P (Q) q_i$ where $Q \triangleq q_1 + q_2$ and $P (Q)$ is the inverse demand function. We assume $P (Q)$ has a finite saturation point $\bar{Q}$ such that $P (\bar{Q}) = 0 \forall Q \geq \bar{Q}$. Besides, $P (Q) \in C^2$ on its domain with $P'(Q) > 0 \forall Q < \bar{Q}$ and $P''(Q) < 0 \forall Q < \bar{Q}$. We do not make any initial assumption about $P''(Q)$ but we establish that $P''(Q) q_i + P'(Q) < 0$ so that quantities are strategic substitutes.

Let $\Pi^i : Q_i \times Q_j \times K_i \to \mathbb{R}$ being $i$’s benefits function with $\Pi^i (q_i, q_j, k_i) = F \triangleq R^i (q_i, q_j) - C^i (q_i, k_i) - F$ where $F$ is a quasi-fixed cost. We assume $F > 0$ for the the entrant and $F = 0$ for the incumbent. Note that, as far as the assumption we have made, $\Pi^i (q_i, q_j, k_i) \in C^2$ on its domain.

We assume Inada-type conditions so that each firm is able to serve alone the market,

$$\lim_{(q_i, q_j) \to (0, 0)} \left[ P (Q) - C^i (q_i, k_i) \right] > 0 \forall k_i$$

and, by the assumptions we have made, it is also true that

$$\lim_{(q_i, q_j) \to (\bar{Q}, 0)} \left[ P (Q) - C^i (q_i, k_i) \right] < 0 \forall k_i$$

### 4 Characterization of market competition and strategic investment decisions

#### 4.1 Market stage characterization

At market stage, the choices regarding strategic investments have been already made. Therefore, at this stage we have a collection of market stage games parametrized by $k_1$ and $k_2$ with $i$’s benefits given by $\Pi^i (q_1, q_2 | k_1, k_2)$

Let $q'_i \wedge q''_i \triangleq \inf \{ q'_i, q''_i \}$ and $q'_i \lor q''_i \triangleq \sup \{ q'_i \lor q''_i \}$ being the meet and the join of $q'_i$ and $q''_i$, respectively. As the strategy space of firm $i$ at this stage is given by $[0, \bar{Q}] \subset \mathbb{R}$, then it is a chain, with the usual order $\geq$, that is, $\geq$ is a binary relation which satisfies completeness, transitive and antisymmetry. These conditions determine $Q_i$ being a sublattice of $\mathbb{R}$ since, as completeness implies reflexivity, it satisfies the weaker condition of being a partially ordered set, in addition to satisfy that for all $\{ q'_i, q''_i \} \subseteq Q_i$ an infimum and a supremum exist $Q_i$. Besides, $Q_i$ is a complete sublattice such that for any nonempty subset $A \subseteq Q_i$, we have that $\inf (A) \in Q_i$ and $\sup (A) \in Q_i$. This occurs as a consequence that $Q_i$ is a compact subset of the real line.$^{10}$

As far as the assumptions we have made, $\Pi^i_{ij} < 0$ with $i \neq j$ so that $i$’s benefit has strictly decreasing differences in $(q_i, q_j)$ and therefore quantities are strategic sub-

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$^{10}$This is valid as in any euclidean space, a bounded sublattice $S \subset \mathbb{R}^n$ is complete if and only if it is a compact set.
stitutes. Morever, $\Pi^i$ is supermodular in $q_i$ in a trivial way as $i$’s space of strategy at this stage is unidimensional and so for a given $q_j$ and $k_i$ it satisfies $\Pi^i (q_i' \wedge q_i'', q_j, k_i) + \Pi^i (q_i' \vee q_i'', q_j, k_i) \geq \Pi^i (q_i', q_j, k_i) + \Pi^i (q_i'', q_j, k_i)$.

These conditions establish a (smooth) submodular market stage game and doing the trick of defining $\tilde{q}_2 \triangleq -q_2$ we can convert it into a (smooth) supermodular market stage game so that both incumbent’s benefits and entrant’s benefits has strictly increasing differences in $(q_1, \tilde{q}_2)$. This allows us to apply the theory of supermodular games\(^\text{11}\) for the pair $(q_i, q_j) = (q_1, \tilde{q}_2)$. Let’s define player $i$’s best-response correspondence as $Q_i^{BR} : Q_j \Rightarrow Q_i$ with $Q_i^{BR} (q_j) \triangleq \arg \max_{q_i} \Pi^i (q_i, q_j | k_i, k_j)$. Then we know that $Q_i^{BR} (q_j)$ is nonempty and has a greatest and least best-response selection for a given $q_j$. Let’s define $q_i^{BR} \in Q_i^{BR}$ as a selection of the best-response correspondence and $q_2^{BR} \triangleq q_i^{BR} (q_2) \times q_2^{BR} (q_1)$ as a best joint response selection. By Topkis’ theorem we know that every selection of the best-response correspondence is increasing in rival’s strategy. Let $\eta_i (q_j)$ and $\eta_j (q_j)$ be the selections consisting of greatest and smallest elements for a given $q_j$, respectively.

By Tarsky’s fixed point theorem we know that the set of pure strategy Nash equilibria is nonempty and that it contains a largest and smallest equilibrium\(^\text{12}\) defined by $\eta_1 \triangleq \eta_1 (\tilde{q}_2) \times \tilde{q}_2 (\tilde{q}_1)$ and $\eta_2 \triangleq q_1 (q_2) \times q_2 (q_1)$. It is important to bear in mind that we have derived these results for strategies $(q_1, \tilde{q}_2)$ and therefore, in the largest (smallest) equilibrium, incumbent’s output is the largest (smallest) among all equilibria and entrant’s output is the smallest (largest) among all equilibria.

Although there could be other equilibria, in this paper we characterize all the results for extremal equilibria. This can be justified in a Milgrom and Roberts’ (1990) fashion, as in supermodular games it is always possible to Pareto order all equilibria. Specifically in this model, we have that the largest (smallest) equilibrium corresponds to the Pareto-best (Pareto-worst) equilibria for the incumbent and Pareto-worst (Pareto-best) equilibria for the entrant.

For comparative statics, note that $K_i$ is also a chain and as a consequence satisfies the weaker condition of being a partially ordered set. Besides, from the assumptions we have made it happens that $\Pi^i_{k_i} > 0$ so that $i$’s benefit has strictly increasing differences in $q_i$ and $k_i$. Then, all best-response selections are increasing in $k_i$ and, in particular, the extremal best-response selections satisfy this condition so that the smallest and largest Nash equilibrium are strictly increasing in the $k_i$. Besides, by the maximum theorem and the fact that $\Pi^i \in C^2$ we know that $q_i^{BR} (\cdot) \in C^1$, so that $\frac{\partial q_i}{\partial k_i} |_{\eta_1} > 0$ and $\frac{\partial q_i}{\partial k_i} |_{\eta_2} > 0$.

\(^{11}\)For an expouse of supermodularity and, in particular of supermodular games, see for instance Topkis (1998).

\(^{12}\)In fact, these coincide with the smallest and largest elements of the set of strategies that survive iterated deletion of strictly dominated strategies and with the bounds of the set of rationalizable strategies of each player. Moreover, as we are dealing with a two players game, both sets are going to be identical.

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for \((q_i, q_j) = (q_1, q_2)\). In addition, as \(\frac{\partial q_i}{\partial k_i} = \frac{\partial q_i}{\partial q_j} \frac{\partial q_j}{\partial k_i}\), then \(\frac{\partial q_i}{\partial q_j} \bigg|_{\mathbb{R}} < 0\) and \(\frac{\partial q_i}{\partial k_j} \bigg|_{\mathbb{R}} < 0\) for \((q_i, q_j) = (q_1, q_2)\). In order to obtain a deeper understanding of these results, note that \(k_i\) only affects the the cost of the \(i\)’s firm. Therefore, it has a direct effect in the benefits of \(i\)’s firm but not on those of \(j\)’s firm as \(\Pi^i_{k_i} = 0\). Therefore, the only impact \(k_i\) has in \(j\)’s benefits is indirect and it occurs through the incentives of \(i\)-th firm to choose a determined level of \(q_i\) which, at the same time, modifies the \(j\)-th firm’s incentives to choose \(q_j\).

Supermodular games allow equilibria being in the boundary of the feasible set for some value of parameters. Nonetheless, by the Inada-type conditions we have assumed, in our market stage game all equilibria must be interior and as a consequence can be characterized by the following first order condition,

\[
\Pi^i = P'(Q) q_i + P(Q) - C_i = 0
\]  

(1)

### 4.2 Strategic investment stages characterization

In this part, we characterise the choice of \(k_i\) made by firms given market stage extremal equilibria. In order to avoid tedious notation, we denote \(i\)’s extremal equilibrium strategy as \(q_i (k_i, k_j)\) and all quantities’ derivatives with respect to strategic investment variables are assumed to be evaluated at these points.

We start deriving the relation between \(k_i\) and \(k_j\) assuming both decisions are made simultaneously. So profit of \(i\)’s firm is given by \(\Pi^i (q_i (k_i, k_j), q_j (k_i, k_j), k_i)\). We assume that for these functions is true that \(\Pi^i_{k_i, k_i} < 0\) for \(i = 1, 2\) so that a best-response function exists which be denoted as \(k^{BR}_i (k_j)\) and satisfies \(k^{BR}_i (k_j) \in C^1\). Besides, we make the assumption that hessian matrix is definite negative so that direct effects dominates indirect ones and we have a unique (and stable) Nash equilibrium.\(^3\) Besides, for the assumptions made over cost functions, we have that this equilibrium is interior.

Differentiating first order condition for \(i\)’s firm and working out the expression, we have that

\[
sg \left( \frac{d k^{BR}_i (k_j)}{d k_j} \right) = sg \left( \frac{d R_i^j}{d k_j} \frac{\partial q_i}{\partial k_i} + \frac{\partial \Pi^i}{\partial q_j} \frac{\partial^2 q_j}{\partial k_i \partial k_j} + \frac{\partial^2 \Pi^i}{\partial k_i \partial q_i} \frac{\partial q_i}{\partial k_j} \right)
\]  

(2)

where \(\frac{d R_i^j}{d k_j} = R_{ij} \frac{\partial q_i}{\partial k_j} + R_{ij} \frac{\partial q_i}{\partial k_j}\). \(^3\) A hessian matrix definite negative is a sufficient condition for the case of two players to have a best response mapping that is a contraction on the entire strategy space so that a unique and globally stable Nash equilibrium is assured.
We define that strategic investments are substitutes or complements whether (2) is negative or positive, respectively. Without additional assumptions, we cannot determine the sign of this term.\textsuperscript{14} We do not make an initial assumption about its sign.

Finally, note that this stage defines a supermodular game as $K_i$ is a compact subset of the real line such that is a complete sublattice of $\mathbb{R}$ and, in case strategic investments are substitutes so that the game is submodular, we can use again the trick that $k_2 = -k_2$ in such a way that it has increasing differences in $(k_i, k_j)$. Moreover, $i$’s benefits are supermodular in $k_i$ as its unidimensional. The relevancy of this fact is that we can apply the property that as there is unique Nash Equilibrium is the only outcome that survives iterated deletion of strictly dominated strategies and as a consequence the stage is dominance solvable.\textsuperscript{15}

5 Firms’ behavior

Given the analysis presented above of how firms makes decisions about its variables, we now proceed to characterize the solutions of the game.

5.1 Entrant’s behavior

The relations we have identified so far enable us to attain some results regarding firms’ decision respect to strategic investment. Let’s start with entrant’s decision. If the entrant decides to enter, given $k_1$, it sets $k_2$ in order to maximize benefits. The decision about $k_2$ is carried out knowing that it alters its incentives to play $q_1$ and, as a consequence, it affects the incumbent’s choice of the pair $q_1$. Then,

$$k_2 (k_1) = \arg \max_{k_2} \Pi^2 (q_1 (k_2), q_2 (k_2), k_2 | k_1)$$

\textsuperscript{14} However, we could expect that $\frac{d k_2^2}{\Pi (k_j)} < 0$ based on the next intuition. We have $\frac{d R_i}{\Pi_j}$ has an undefined sign as $\frac{d R_i}{\Pi_j} = R_{ij} \frac{\partial q_i}{\partial k_j} + R_{ijk} \frac{\partial q_i}{\partial k_j}$.

If we made the assumption that $\Pi'' (Q) > 0$, it would determine that $\frac{d R_i}{\Pi_j} > 0$.

In that case:

$$\text{sg} (\frac{d k_2}{\Pi_j}) = \text{sg} \left( \frac{d R_i}{\Pi_j} \frac{\partial q_i}{\partial k_j} + R_{ij} \frac{\partial q_j}{\partial k_i} \frac{\partial q_i}{\partial k_j} + R_{ijk} \frac{\partial q_j}{\partial k_i} \frac{\partial q_i}{\partial k_j} \right)$$

If the term $\frac{\partial^2 q_i}{\partial k_i \partial k_j}$ is neither negative nor too strong, the sign of $\frac{d k_2}{\Pi_j}$ would be negative.

\textsuperscript{15} A game is dominance solvable in case the set of strategies remaining after iterated elimination of strictly dominated strategies is a singleton.
We assume that second derivate is negative so a solution exists and is unique. What
is more, the solution can be obtained from the first order condition,
\[
\frac{d\Pi^2}{dk_2} = \frac{\partial \Pi^2}{\partial q_1} \frac{\partial q_1}{\partial k_2} - C'_{k_2} = 0
\]
(3a)

where we used the fact that \( \frac{\partial \Pi^2}{\partial q_2} = 0 \). Let’s define \( SE_2 \triangleq \frac{\partial \Pi^2}{\partial q_1} \frac{\partial q_1}{\partial k_2} \) as the strategic
effect of \( k_2 \). The solution to (3a) will be denoted as \( k_2 (k_1) \) and \( k_2^A \) will stand for the
level of \( k_2 \) for the optimal \( k_1 \).

5.2 Incumbent’s behavior

At the first stage, the incumbent has to decide about the level of its strategic investment.
The decision about \( k_1 \) is done knowing that it alters its incentives to play \( q_1 \) and, as a
consequence, it affects the entrant’s choice of the pair \( (k_2, q_2) \). The optimal \( k_1 \) is given by,

\[
k_1^* \triangleq \arg \max_{k_1} \Pi^1 [q_1 (k_1, k_2 (k_1)), q_2 (k_1, k_2 (k_1)), k_1]
\]

According to the assumptions we have made, a solution exists and is unique. Notwithstanding,
unlike entrant’s strategy, incumbent’s behavior displays three different strategies in equilibrium depending on the level of entrant’s quasi-fixed cost.

In the first place, for low levels of \( F \), the incumbent accommodates entry. This implies that, although the entrant anticipates a profitable entry, by using \( k_1 \), the incumbent improves its position relative to the entrant.

In the second place, for intermediate values of \( F \), the incumbent can use \( k_1 \) to make
the entry unprofitable. Through an adequate choice of \( k_1 \), the incumbent is capable of
keeping entrant’s postentry profits low enough to remain as a monopolist.

It has to be considered an additional possibility as, for high values of \( F \), the incumbent
could behave as a monopolist so that it does not take into account any strategic
consideration and yet entry would not occur. In that case, it is said that entry is
blockaded.

5.2.1 Blockaded entry

Before deepen into the strategic analysis of the game, we consider the situation where
entry is blockaded and, in consequence, the incumbent acts as a monopolist. If it
behaves in that fashion, its problem is,
\[
\max_{q_1, q_2, k_1} \Pi^1 (q_1, q_2, k_1) = P (Q) q_1 - C^1 (q_1, k_1)
\]

The first order condition for strategic investment is given by,

\[
C^1_{k_1} = 0
\]

We denote the solution as \( k^M_1 \) and the benefits associated for an optimal level of monopolist quantities as \( \Pi^1 (k^M_1) \).

### 5.2.2 Entry accommodation

In case the incumbent let the entrant enters, it chooses the level of strategic investment that solves,

\[
\max_{k_1} \Pi^1 [q_1 (k_1, k_2 (k_1)), q_2 (k_1, k_2 (k_1)), k_1].
\]

subject to \( \Pi^2 [q_1 (k_1, k_2 (k_1)), q_2 (k_1, k_2 (k_1)), k_2 (k_1)] - F \geq 0 \)

The first order condition determines,

\[
\frac{d\Pi^1}{dk_1} = \frac{\partial \Pi^1}{\partial q_2} \frac{\partial q_2}{\partial k_1} + \frac{\partial \Pi^1}{\partial k_2} \frac{dk_2}{dk_1} - C^1_{k_1} = 0
\]

Where we used the fact that \( \frac{\partial \Pi^1}{\partial q_2} = 0 \). We denote this solution as \( k^A_1 \). The term \( SE_1 \triangleq \frac{\partial \Pi^1}{\partial q_2} \frac{\partial q_2}{\partial k_1} \) gives the strategic effect of \( k_1 \) at the market moment and is similar to the one obtained for the entrant’s choice of \( k_2 \). Notwithstanding, the incumbent has an additional term because the \( k_1 \) chosen is seen by the entrant before this decides about \( k_2 \). We coin the term "Strategic Investment Effect" for \( SE_1 \triangleq \frac{\partial \Pi^1}{\partial q_2} \frac{\partial q_2}{\partial k_1} \).

### 5.2.3 Entry deterrence

Let’s consider the case where the incumbent chooses a level of \( k_1 \) in order to deter entry. We denote it as \( k^D_1 \) and it is obtained from the following optimization problem,

\[
\max_{k_1} \Pi^1 [q_1 (k_1), k_1]
\]
subject to $\Pi_2^2 [q_1 (k_1, k_2 (k_1)), q_2 (k_1, k_2 (k_1)), k_2 (k_1)] - F \leq 0$

The qualitative pattern of incumbent’s strategy to make entry unprofitable can be established taking the total derivative of $\Pi^2(\cdot)$ with respect to $k_1$,

$$\frac{d \Pi^2}{dk_1} = \frac{\partial \Pi^2}{\partial q_1} \frac{\partial q_1}{\partial k_1} + \frac{\partial \Pi^2}{\partial q_2} \frac{\partial q_2}{\partial k_1} + \frac{\partial \Pi^2}{\partial k_1} \frac{\partial k_1}{\partial k_1}$$

Where we used the fact that $\frac{\partial \Pi^2}{\partial q_2} = 0$. Using (3a), it can be demonstrated that the expression is equal to,

$$\frac{d \Pi^2}{dk_1} = \frac{\partial \Pi^2}{\partial q_1} \frac{\partial q_1}{\partial k_1} \leq 0$$

(6)

This implies that increments in $k_1$ reduce monotonically entrant’s benefits. The importance of this result lies in the fact that in order to characterize deterrence strategy in a qualitative way there is no need to know whether strategic investments are substitutes or complements. It only depends on the effect of the strategic investment in tactical variable’s marginal benefit (if the incumbent is tough or soft, in Fudenberg and Tirole’s (1984) terminology) and how the tactical variable affects entrant’s benefits. Then, although now both firms can make strategic investments, the qualitative feature of the entry deterrence strategy resembles the one derived in Fudenberg and Tirole’s (1984) two-stage model. We also can establish another feature of the entry deterrence strategy.

**Lemma 1** The optimal level of strategic investment that deters entry satisfies that the entrant breaks even.

**Proof.** See appendix. ■

6 Strategic benchmark

The comparisons we present in both this and next section are defined for incumbent’s strategic investment decision. In addition, as it is standard in the literature, we only deal with accommodating and deterrence strategies as those are the only situations involving strategic considerations. In this section, following Fudenberg and Tirole (1984) we compare the incumbent’s accommodation solution with the solution defined by the
open loop equilibrium\textsuperscript{16} and we do the proper with incumbent’s entry deterrence solution against monopoly solution. The open loop solution is defined by a game where at first stage firms decides simultaneously about strategic investment variables and then, after seeing the decisions which have been made, both firms decide about quantities. This enables us to isolate the strategic value of being an incumbent. We refer to the incumbent solution as the closed loop solution. Unlike two-stage model, the key term to obtain overinvestment is the Strategic Investment Effect. For this reason, we have to assume a sign for strategic investment best-response’s slope. Specifically we need to assume

\[
\frac{dk_i^{BR} (k_j)}{dk_j} < 0 \tag{7}
\]

Also, as we stated above, all results are shown for extremal equilibria of the market stage game.

6.1 Comparison respect to the open loop equilibrium

We start with the comparison of the accommodation solution against the open loop solution Let’s start characterizing the open loop solution. Given one of the extremal Nash equilibria at the market stage, each firm sets \( k_i^{OL} \) such that,

\[
k_i^{OL} \triangleq \arg \max_{k_i} \Pi^i (q_i (k_i, k_j), q_j (k_i, k_j), k_i)
\]

As far as the assumptions we have made, a unique interior solution exists and it satifies,

\[
C_{k_i}^1 = \frac{\partial \Pi^1}{\partial q_2} \frac{\partial q_2}{\partial k_i}
\]

**Proposition 1** In case competition is in quantities and quantities as well as strategic investments are strategic substitutes, the incumbent chooses a level of strategic investment in accommodation that is higher than the level of strategic investment in the open loop equilibrium where strategic investments are chosen simultaneously by firms and quantity decisions are made after these.

\textsuperscript{16}In a general way, the term open loop refers to an information structure of a multi-stage game where players cannot observe the play of their opponents (Fudenberg and Tirole, 1991). This implies that players’ strategy cannot be contingent on the actual choice of its opponents. It is usually used as a benchmark because it allows to measure strategic effects as we can compare how moving from a situation where strategies cannot be used in a strategic as they are unobservable to one where they are used in that way, accounting for the influence on opponent’s actions, affects player’s strategies.
Proof. See Appendix. ■

6.1.1 Comparison respect to monopoly solution

We proceed to make the comparison of the entry deterrence solution against the monopoly solution. The latter has the same characterization that the solution that emerges in case entry is blockaded.

Proposition 2 In case competition is in quantities and quantities as well as strategic investments are strategic substitutes, the incumbent chooses a level of strategic investment in entry deterrence solution that is higher than the level of strategic investment chosen by a monopolist.

Proof. See Appendix. ■

6.2 First-mover advantage

From the result we derived above, it is possible to demonstrate that in this kind of model, and in case (7), a first-mover advantage exists as in Fudenberg and Tirole (1984). What is more, an in accordance with Ware’s (1984) result17, we demonstrate that this difference is less than in a two-stage specification.

Proposition 3 In case competition is in quantities and strategic investments as well as quantities are strategic substitutes, it exists a first-mover advantage. Moreover, this is diminished respect to a two-stage model.

Proof. See Appendix. ■

7 Aggregate Industry Benchmark

We consider the existence of a regulator whose aim is to maximize aggregate benefits. The regulator is constrained to a second-best framework where it is only able to select the level of each firm’s strategic investment. Once this is done, firms voluntarily decide whether they enter or not in the market. If both decide to participate, they compete in quantities. In case, just one firm decides to enter, it selects quantities as a monopolist.

17Although Ware (1984) arrives to the same result, this is only shown for the case of a linear demand and constant marginal costs.
7.1 Additional assumptions for market stage

We make an additional assumption which modifies the equilibria of the market stage. We assume that \( C_i^i (q_i, k_i) < 0 \) \( \forall q_i, k_i \) so that cost function is convex. It can be demonstrated that this assumption and the fact that demand is monote decreasing implies that,

\[
\left| \frac{d \Pi_i^R (q_j)}{d q_j} \right| = \left| \frac{\Pi_{ij}}{-\Pi_{ii}} \right| < 1
\]  

(8)

The assumption (8) is a sufficient condition for having a hessian matrix of both firms’ benefits that is definite negative. As a consequence, we have a unique (and stable) Nash equilibrium at the market stage. Besides, as the game remains supermodular, we know that this is the unique outcome that survives iterated deletion of strictly dominated strategies so that market stage becomes dominance solvable.

7.2 Regulator’s maximization problem

Mathematically, the regulator’s problem is given by,

\[
\max_{k_1, k_2} \Pi = \mathbf{1} (\Pi^1 (\cdot) > 0) \mathbf{1} (\Pi^2 (\cdot) - F > 0) [P (Q) (q_1 + q_2) - C^1 (q_1, k_1) - C^2 (q_2, k_2) - F] + \\
\mathbf{1} (\Pi^1 (\cdot) > 0) (1 - \mathbf{1} (\Pi^2 (\cdot) - F > 0)) [P (Q) (q_1) - C^1 (q_1, k_1)]
\]

subject to \[
\begin{aligned}
q_1 &= \mathbf{1} (\Pi^1 (\cdot) > 0) q_1 (k_1, k_2) \\
q_2 &= \mathbf{1} (\Pi^2 (\cdot) - F > 0) q_2 (k_1, k_2)
\end{aligned}
\]

Where \( \mathbf{1} (\cdot) \) is an indicator function. We denote \( k^C \overset{\Delta}{=} (k_1^C, k_2^C) \) to the solution to the optimization problem (the superscript \( C \) stands for "collusion"). We assume that the hessian matrix is definite negative so that a global unique solution exists.

Let’s analyze some features of this optimization problem. In first place, suppose firms compulsory enter in the market by regulator’s choice. In that case, regulator’s decision would consist of letting both firms producing or just the incumbent staying alone in the market. In other terms, the entrant producing alone would not be optimal because of its entering cost. Furthermore, the optimal choice requires evaluating the relative benefits concerning these options. On the one hand, there are advantages in having both firms producing as the cost function is convex. On the other hand, unlike letting the incumbent staying alone in the market, if the entrant remains in the market an entering cost has to be bear while at the same time there is a loss of aggregate benefits due to competition in quantities.

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To expose regulator’s problem solution, we start characterizing the two solutions we mentioned above. Then, we establish further assumptions. Later, we proceed to characterize the solution with voluntary participation. Finally, we make some comments on welfare benchmarks.

7.3 Characterization of solutions

Let’s start with the case where both firms produce. Note that as firms are symmetric (except for the entering cost) and cost function is convex, the maximum aggregate benefits are obtained when \( k_1^C = k_2^C \). Then, the first order condition for \( k_i \) is,

\[
C^i_{k_i} = \frac{\partial \Pi^i}{\partial q_i} \frac{\partial q_i}{\partial k_i} + \frac{\partial \Pi^i}{\partial q_j} \frac{\partial q_j}{\partial k_i}
\]

where we used the fact that \( \frac{\partial \Pi^i}{\partial q_i} = 0 \) for \( i = 1, 2 \).

This gives us as result,

\[
C^i_{k_i}(q_i, k_i) = P'(Q) q_i \left( \frac{\partial q_i}{\partial k_i} + \frac{\partial q_i q_j}{\partial k_j \partial k_i} \right)
\]  \hspace{1cm} (9)

Also, it happens that \( q_1 (k_1^C, k_2^C) = q_2 (k_1^C, k_2^C) \). We denote this quantities as \( q_i^C \). Then, evaluated at the equilibrium point, we have that,

\[
C^i_{k_i}(q_i^C, k_i^C) = P'(Q^C) q_i^C \left( \frac{\partial q_i}{\partial k_i} \bigg|_{k_i^C} + \frac{\partial q_i q_j}{\partial k_j \partial k_i} \bigg|_{k_i^C} \right)
\]  \hspace{1cm} (10)

As \( \frac{\partial q_i}{\partial k_i} = \frac{d q_i}{d q_i} \frac{d q_i}{d k_i} \) and (8), it always happens that \( \frac{\partial q_i q_j}{\partial k_j \partial k_i} > 0 \).

On the other hand, regarding the case where the incumbent produces alone, the solution is given by (4a) as it coincides with the situation of blockaded entry.

7.4 Additional assumptions

Suppose entry is compulsory. Then the regulator’s choice between depend on the level of the entrant’s quasi-fixed cost. The problem can be thought as a benevolent regulator, which wants to maximize aggregate benefits and it has to choose between two options: letting the incumbent producing alone or buying entrant’s production technology for an \( F \) price. If the price of this technology is cheap enough, the latter would be profitable as, given that both technologies are convex, it could supply same quantities with a lower cost.

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As we do not have any quantitative information about relative benefits, we need to make further assumptions. The assumptions we make are done in order to reflect cases where the strategic investment variables are payoff-relevant. Specifically, we establish two assumptions and two corollary.

In the first place, we assume

\[ \Pi^1 (k^C_1, k^C_2) + \Pi^2 (k^C_1, k^C_2) > \Pi^1 (k^M_1) + \Pi^2 (k^A_1, k^A_2) \]  
(11)

Moreover, a corollary of this is

\[ \Pi^1 (k^C_1, k^C_2) + \Pi^2 (k^C_1, k^C_2) > \Pi^1 (k^M_1) \]  
(12)

We can provide an interpretation of both inequalities taking as starting point the analysis of (11). This translates into the convexity of production technology is more determining in aggregate benefits than the loss of benefits due to competition in quantities. If monopoly gave more benefits, it would imply that, even colluding in strategic investment variables, the increment of aggregate benefits would disappear because of competition at the market stage by quantities. This, in turn would imply that regulating strategic investment variables would not be as relevant as setting each firm’s level of production.

Once we assume (11), in case entry is compulsory, regulator’s decision depends exclusively on the level of entering cost: for low levels of \( F \) it would choose the collusive solution and for high levels of \( F \) it would prefer that the incumbent remains as a monopolist. Note that (12) reinforces the idea that production technology is convex enough or, alternatively, it could be interpreted as establishing a high value of being an incumbent so that, without taking into account fixed costs, entrant’s benefits are low enough that does not reverse the order of (12). Also, notice that as in the monopoly’s solution the benefits of any individual firm are maximized it is true that \( \Pi^2 (k^C_1, k^C_2) > \Pi^2 (k^M_1) \) and as a consequence, (11) implies that

\[ \Pi^2 (k^C_1, k^C_2) > \Pi^2 (k^A_1, k^A_2) \]  
(13)

\[ ^{18} \text{Alternatively, there would be no need to assume (12) and (11) if we had established that } \Pi^2 (k^C_1, k^C_2) - \Pi^2 (k^A_1, k^A_2) > \Pi^2 (k^M_1) - \Pi^2 (k^C_1, k^C_2). \text{ It can be demonstrated that using the fact that } \Pi^1 (k^C_1, k^C_2) = \Pi^2 (k^M_1, k^M_2) \text{ and } \Pi^1 (k^M_1) = \Pi^2 (k^M_2), \text{ they are implied by that inequality. This could be interpreted as that the increment in benefits resulting from a regulator which implements the collusive solution when both firms are competing in the market is bigger than the increment in benefits resulting of a situation where a regulator leaves the collusive solution and let the incumbent remains as a monopolist.} \]

\[ ^{19} \text{In case strategic investments are substitutes } \Pi^2 (k^C_1, k^C_2) > \Pi^2 (k^A_1, k^A_2) \text{ holds even without the need to assume (11). This is because we know that } \Pi^1 (k^C_1, k^C_2) + \Pi^2 (k^C_1, k^C_2) > \Pi^1 (k^A_1, k^A_2) + \Pi^2 (k^A_1, k^A_2) \text{ as aggregate benefits in } (k^C_1, k^C_2) \text{ are maximized and the solution is unique. Besides,} \]
The other assumption is,

\[ \Pi^2(k_1^M, k_2(k_1^M)) > \Pi^2(k_1^C, k_2^C) \]  

(14)

This means that entrant’s benefits when an incumbent choose \( k_1^M \) and entry is not blockaded are bigger than entrant’s collusive benefits.

### 7.5 Solution characterization with voluntary entry

We proceed to analyze the situation where the entrant voluntarily decides whether enters or not. The main conclusion is that the solution a regulator would choose in case entry is compulsory can be in fact implemented.

The monopoly solution is always feasible as the regulator always can ask for a level of \( k_2 \) so that entrant anticipates negative benefits. Hence, it only remains demonstrating that collusive solution is feasible when it is optimal for the regulator to choose it.

**Lemma 2** For the range of \( F \) values where collusive solution gives more aggregate benefits than the monopoly solution, the entrant anticipates positive benefits

**Proof.** See Appendix. ■

### 7.6 Optimal solutions according to different levels of entering cost

In order to make a comparison, we need to confront the regulator’s solution with the closed loop solution that prevails for a given level of \( F \).

**Lemma 3** For the range of values of \( F \) where accommodation is the incumbent’s optimal strategy, the regulator chooses the collusive solution

**Proof.** See Appendix. ■

Note that this also implies that for a level \( F+\varepsilon \), where \( F \) determines that \( \Pi^1(k_1^A, k_2^A) = \Pi^1(k_2^D) \) and \( \varepsilon \) is small, it happens that \( \Pi^1(k_1^A, k_2^A) < \Pi^1(k_2^D) \) and \( \Pi^1(k_1^C, k_2^C) + \Pi^2(k_1^C, k_2^C) - F > \Pi^1(k_1^M) \) by continuity of solutions.

It remains to determine what happens with respect to incumbent’s optimal decision when \( F \) increases even more and the regulator chooses the monopoly solution.

\[ k_1^A > k_1^A \]  

and \( \frac{d\Pi^1}{dx_1} < 0 \) \( \forall k_1, k_2 \), it happens that \( \Pi^1(k_1^A, k_2^A) > \Pi^2(k_1^A, k_2^A) \). Also, as firms are symmetric, \( \Pi^1(k_1^C, k_2^C) = \Pi^2(k_1^C, k_2^C) \). Adding this, we have that \( \Pi^2(k_1^C, k_2^C) - \Pi^2(k_1^A, k_2^A) > 0 \)  

\[ n^2(x_1^A, x_2^A) - n^2(x_1^C, x_2^C) > 0. \]
**Lemma 4** For the range of values of $F$ where entry is blockaded, the regulator chooses the monopoly solution.

**Proof.** See Appendix. □

Summarizing the results in a diagram form,

![Diagram](image)

Figure 1

### 7.7 Solutions comparison with substitute strategic investments

It is shown that from the comparison between market solution and its appropriate regulator’s solution, it can be concluded that it exists overinvestment of the incumbent’s strategic investment. As we did previously with the strategic benchmark, we will just compare the solutions in case the incumbent either accommodates or deters entry.

**Proposition 4** In case competition is in quantities and strategic investments as well as quantities are strategic substitutes, incumbent’s accommodation strategy implies overinvestment respect to the solution that maximizes aggregate benefits which, for that level of $F$, is the collusive one.

**Proof.** See Appendix. □

**Proposition 5** In case competition is in quantities and these are strategic substitutes, for the range of $F$ values where incumbent deters entry, it exists overinvestment respect to the solution that maximizes aggregate benefits which coincides for a range of $F$ with the collusive one and for other range of $F$ with monopoly.

**Proof.** See Appendix. □
7.8 Solutions comparison with complementary strategic investments

It is important to note that for the previous comparison, while entry accommodation required to assume that strategic investments are substitutes, the entry deterrence strategy comparison does not. This is because the entry deterrence strategy is the same irrespective of the sign of strategic investment best-response’s slope. For this reason, in order to demonstrate that overinvestment exists, it only rest to obtain the result for for the case where the incumbent accommodates entry. With this aim, we need to make a further assumption which is \( \frac{d^2 q_2}{d k_1^2} < 1 \).

**Proposition 6** In case competition is in quantities and these are strategic substitutes while strategic investments are complements, then if \( \frac{d^2 q_2}{d k_1^2} < 1 \), incumbent’s accommodation strategy implies overinvestment respect to the collusive solution

**Proof.** See Appendix. ■

7.9 Welfare considerations

In case we consider an alternative objective function for the regulator such that gives a weight \( \theta \in (0, +\infty) \) to consumer surplus\(^{21}\), the objective function would be,

\[
W \triangleq \Pi + \theta CS
\]

where \( CS \triangleq \int P(z) dz - P(Q(k_1, k_2)) Q(k_1, k_2) \) and \( Q(k_1, k_2) \triangleq q_1(k_1, k_2) + q_2(k_1, k_2) \).

Let’s analyze this case irrespective of the solution that the regulator would prefer to implement. This type of representation allows us to do comparative statics for the parameter \( \theta \) in a straightforward way as the game remains supermodular. In order to derive the relation between \( k_i \) and \( \theta \), we need to determine the sign of the cross derivative,

\[
\frac{\partial^2 W}{\partial \theta \partial k_i} = -P'(Q(k_1, k_2)) Q(k_1, k_2) \left( \frac{\partial q_i}{\partial k_i} + \frac{\partial q_j}{\partial k_i} \right)
\]

(15)

As \( \frac{\partial q_i}{\partial k_i} = \frac{\partial q_i}{\partial \theta} \frac{\partial \theta}{\partial k_i} \) and (8), it is true that \( \frac{\partial q_i}{\partial k_i} + \frac{\partial q_j}{\partial k_i} > 0 \). This means that (15) is positive so that the bigger the consumer surplus’s weight the higher the level of strategic

\(^{20}\)This assumptions is standard and it is a sufficient condition to have a hessian matrix definite negative.

\(^{21}\)Income effects that could turn the consumer surplus inaccurate are ignored.
investment a regulator would prefer. Therefore, by continuity of solutions we know that for small levels of \( \theta \) the solution we found above prevails but as \( \theta \) increases the baseline level of strategic investment also increases and as a consequence it exists a \( \theta^* \) critical level where the overinvestment pattern does not exist anymore. However, that is all we can say about it as, without strong further assumptions, we cannot give more precision about the characteristics of \( \theta^* \).

8 Concluding Remarks

In the present paper we have characterized three-stage entry deterrence models with quantity market competition under the premise that overinvestment is not a general result. For this reason, our focus has been to specify relevant baselines for which overinvestment pattern still works as an accurate description of the model, in addition to interpret their consequences in the context where the result takes place. While strategic benchmark is interesting in itself, as it allows us to isolate the strategic effect and value of being an incumbent, it has a narrow range of application as far as market intervention concerns. In a lot of cases, regulator’s objective could be maximizing aggregate benefits of an industry. For exporting industries, this would be the case because, as firms sell sell its product abroad, consumer’s welfare is not relevant. Besides, as we consider an industry with non-negligible entering cost, other strategies than accommodation can be displayed in equilibrium. Surprisingly, we show that to arrive to the overinvestment result there is no need to take into consideration the sign of strategic investment best-response function’s slope (that is, strategic investment variables being strategic substitutes or complements). What is more, although we have considered only a strategic variable that reduces marginal costs, this approach is useful even for more general instruments. As we have already stated, the consideration of a particular strategic investment was for the sake of simplicity but, actually, all results hold for any strategic investment that increases marginal benefit of tactical variable (v. gr., capacity, sales effort).

One important consideration for future research that we did not address in this paper is uncertainty. In our model, the strategic investment decision takes the form of expenditure in R&D on production technology which reduces marginal costs with certainty. Behind, it lies the assumption that the innovative process is deterministic. However, this is far from being the case and increments in R&D only has an impact on the probability of succeeding. Therefore, firms as well as regulator would take into account other considerations at the moment of choosing the optimal level of strategic investment. Then, same reasoning applies in case, for instance, of advertising.
References


Appendix

**Proof of Lemma 1.** As (6) holds, in case $k_{1}^{M} \geq k_{1}^{D}$ the incumbent would choose $k_{1}^{M}$ and entry would be blocked. So suppose $k_{1}^{M} < k_{1}^{D}$ and assume $k_{1}^{D}$ is actually the solution to incumbent’s optimization problem. If entrant decides to enter to the market, it receives $\Pi^{2}(\cdot) - F < 0$. By Inada-type conditions and strict concavity of benefits function, we know that $\frac{d\Pi^{1}}{dk_{1}} \bigg|_{k_{1}^{D}} < 0$. Besides, we know that entrant’s strategic investment and extremal equilibria are continuous in $k_{1}$. From this, it follows that $\Pi^{2}$ is also continuous in $k_{1}$. Therefore, it is possible to find a $k_{1}'$ near from $k_{1}^{D}$ which satisfies $\Pi^{2}(\cdot) - F < 0$ and gives more benefits. But then $k_{1}^{D}$ cannot be optimal. ■

**Proof of Lemma 2.** It suffices to demonstrate that for the $F$ level such that $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F = \Pi^{1}(k_{1}^{M})$ then $\Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F < 0$. Suppose not, so that $\Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F < 0$. Then in order to the sides being equal it must happen that $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) > \Pi^{1}(k_{1}^{M})$. However, this cannot be possible as $\Pi^{1}(k_{1}^{M})$ are the highest benefits a firm can perceive by its own. ■

**Proof of Lemma 3.** For a level of $F = 0$, we know that the incumbent would accommodate entry while, by (12), the regulator would choose the collusive solution. Then we proceed to determine what happens as $F$ increases. In particular, we assume that $F$ is high enough so that $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) = \Pi^{1}(k_{1}^{M})$. In that point, necessarily $\Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F > 0$ because if $\Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F < 0$ then the incumbent would either deter or blockade entry. Now, suppose that $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F < \Pi^{1}(k_{1}^{M})$. Therefore, $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C}) < \Pi^{1}(k_{1}^{M}) + F$ which also implies that $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C}) < \Pi^{1}(k_{1}^{M}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C})$. But this contradicts (11). ■

**Proof of Lemma 4.** Suppose $F$ increases until the point where $\Pi^{1}(k_{1}^{C}, k_{2}^{C}) + \Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F = \Pi^{1}(k_{1}^{M})$. We have already demonstrated that for that level of $F$, the incumbent does not accommodate entry. Besides, if it is feasible, it would prefer to blockade entry than deter entry. But at that point we have demonstrated that $\Pi^{2}(k_{1}^{C}, k_{2}^{C}) - F > 0$ and, with (14), it determines that $\Pi^{2}(k_{1}^{M}, k_{2}(k_{1}^{M})) - F > 0$ so that the incumbent’s optimal choice is to deter entry. ■

**Proof of Proposition 1.** In case we evaluate first order conditions at the open loop solution we have that $\frac{d\Pi^{1}}{dk_{1}}\bigg|_{k_{OL}} = \frac{\partial \Pi^{1}}{\partial k_{1}} + \frac{\partial \Pi^{2}}{\partial k_{1}} - C_{1} = 0$ for the open loop first order condition and $\frac{d\Pi^{1}}{dk_{1}}\bigg|_{k_{OL}} = \frac{\partial \Pi^{1}}{\partial k_{2}} + \frac{\partial \Pi^{2}}{\partial k_{2}} - C_{1} = 0$ for the closed loop first order condition. This gives an incentive to increase $k_{1}$ in the closed loop solution. This is the direct effect. At the same time, the increase in $k_{1}$ raises indirect effects. However, as we are assuming that benefits are strictly concave in $k_{1}$, direct effects dominate indirect effects so the process converges to a new equilibrium such that $k_{1}^{A} > k_{1}^{OL}$. ■

**Proof of Proposition 2.** If the incumbent is deterring entry, entry is not blocked so that $\Pi^{2}(k_{1}^{M}, k_{2}(k_{1}^{M})) - F > 0$. But then, for (6), it must be happening that $k_{1}^{D} > k_{1}^{M}$. ■
Proof of Proposition 3. It is sufficient for the demonstration of a first-mover advantage that incumbent’s benefits in accommodation are not going to be lower than the open loop benefits. This is so, as $k_1^{OL}$ is feasible so the incumbent will always be at least as well off as in the open loop equilibrium. In fact, if we evaluate first order conditions with respect to strategic investment variable at open loop solution it happens that $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^{OL}} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} \frac{\partial k_2}{\partial k_1} > 0$ so benefits are going to be strictly higher by continuity of solutions. Moreover, for (6), and the fact that $k_1^{OL} < k_1^A$, it happens that entrant’s benefits are lower respect to the open loop solution. This demonstrates that it exists a first-mover advantage. In order to demonstrate that the first-mover advantage is diminished note that, given $k_1$, entrant’s benefits are given by $\Pi^2(q_1(k_2), q_2(k_2))k_1$. A two-stage specification is similar to a three stage model with $k_2 = 0$. Noticing that $\frac{\partial u^1}{\partial k_2} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} < 0$ and the fact that in accommodation $k_2 > 0$, the incumbent’s benefits are lower respect to an entrant which set $k_2 = 0$. Furthermore, entry deterrence is less valuable too because with $k_2 > 0$, entry deterrence strategy requires to increase $k_1$ even more.

Proof of Proposition 4. If the incumbent’s optimal choice is to accommodate entry then the appropriate benchmark is the collusive solution. In case we evaluate first order conditions at the collusive solution we have that $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} \frac{\partial k_2}{\partial k_1} - C_{k_1}^1 = 0$ for aggregate benefits while $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} \frac{\partial k_2}{\partial k_1} - C_{k_1}^1 > 0$ for closed loop solution. As a consequence, it exists an incentive to increase $k_1$, triggering indirect effects. Nonetheless, as benefits are strictly concave in $k_1$, direct effects dominate indirect effects so the process converges to a new equilibrium such that $k_1^A > k_1^C$.

Proof of Proposition 5. The proof requires to demonstrate that both $k_1^D > k_1^C$ and $k_1^D > k_1^M$. For the range of $F$ values where the regulator chooses the collusive solution, in case the incumbent chooses $k_1^D$ it happens that the entrant breaks even while in the collusive solution the entrant has positive benefits. Then, for (6), it must happen that $k_1^D > k_1^C$. Now, suppose we are at the range of $F$ values where the regulator chooses the monopoly solution and the incumbent deters entry. Then, entry would not be blockaded and, for the pair $(k_1^M, k_2(k_1^M))$, it happens that $\Pi^2(k_1^M, k_2(k_1^M)) - F > 0$. By (6), it implies that in order to deter entry it must happen that $k_1^D > k_1^M$.

Proof of Proposition 6. If the incumbent’s optimal choice is to accommodate entry then the appropriate benchmark is the collusive solution. As for the accommodating strategy we have that $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} \frac{\partial k_2}{\partial k_1} - C_{k_1}^1$ while in the collusive solution $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} = \frac{\partial u^1}{\partial q_2} \frac{\partial q_2}{\partial k_2} \frac{\partial k_2}{\partial k_1} - C_{k_1}^1$. If we evaluate both conditions at $k_1^C$ then, if $\left| \frac{\partial k_2}{\partial k_1} \right| < 1$, it happens that $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} = 0$ and $\frac{\partial u^1}{\partial k_1} \bigg|_{k_1^C} > 0$. As a consequence, it exists an incentive to increase $k_1$ which in turn triggers indirect effects. Nonetheless, as benefits are strictly concave in $k_1$, direct effects dominate indirect effects so the process converges to a new equilibrium such that $k_1^A > k_1^C$.■