Bank Runs and Inequality

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Abstract

In this paper, I examine the relationship between income inequality and bank runs. Analyzing data for 17 countries between 1880 and 2013, I find a positive (and statistically significant) correlation between income inequality and the likelihood of bank runs. I propose a banking model to explore the mechanism underpinning this correlation. This model predicts that rising inequality increases the probability of a bank run. Furthermore, I find that income inequality increases consumption allocations in equilibrium, as they depend on the aggregate level of endowment, and the bank can redistribute between depositors, leading to a higher risk in the bank's investment portfolio.

1 Introduction

Financial crises have negative effects on consumption and output (Jensen and Johannesen, 2017, Romer and Romer, 2017), investment, productivity, employment (Chodorow-Reich, 2014), and health (Cutler et al., 2002), among others. Understanding what causes financial crises or how they can be anticipated can help policymakers to act to prevent crises and their effects on economic and social outcomes. In this paper, I look at the relationship between income inequality, bank runs, and financial crises.

In the 1960s, Friedman and Schwartz (1963) recognized bank runs as triggers for financial crises. More recently, Kirschenmann et al. (2016) and Paul (2022) identify income inequality as a significant predictor of financial crises in developed countries. In addition, Malinen (2016) documents that the relationship between income inequality and financial crises operates through the bank credit channel: An increase in income inequality leads to a rise in bank credit or leverage, amplifying credit cycles (which in turn can generate bank runs and then financial crisis). Understanding the correlation between income inequality and bank runs and describing the underlying mechanism can provide valuable insights into the determinants of the financial crisis and overall system fragility.

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This paper addresses two questions. The first is related to the correlation between income inequality and bank runs. To answer this question, I conducted an econometric analysis that suggested a positive correlation between increasing income inequality and the likelihood of bank runs. Specifically, an increase in inequality by 1 standard deviation is associated with a 1 percentage point increase in the probability of a bank run. The unconditional probability of a bank run was estimated to be around 4% for the sample. Thus, a percentage point increase is a rather significant increase. This result holds accounting for different covariates.

The second question is focused on studying the mechanism that underpins the connection between income inequality and bank runs. Understanding how income inequality influences the occurrence of bank runs is crucial for developing targeted policies to address this issue. To answer this question, I extend the bank run model proposed by Allen and Gale (1998) to accommodate heterogeneity in the endowment levels between two groups of agents (otherwise, the groups are ex-ante identical in preferences and mass). More precisely, I have a mean preserving distribution of endowment between groups, where the wedge between endowment levels describes the income inequality of the economy. Additionally, I do not determine ex-ante that payouts are equal between groups of depositors, but nothing prevents the bank from offering this type of contract to the depositors in equilibrium. This model considers only fundamental bank runs (i.e., those that are not produced by sunspots or self-fulfilling prophecies as coordinating mechanisms). The model incorporates the assumptions on preferences and timing from Diamond and Dybvig (1983), but it distinguishes in two features: i) it does not assume a sequential withdrawal by the agents, and ii) illiquid assets held by the banks are risky and perfectly correlated across banks.

The model has three main theoretical results. First, the depositors' payouts at any period are equal in equilibrium. This is because both payouts depend on the economy's aggregate wealth, not the distribution between agents. Second, allowing the liquidation value before the maturity of the risky asset to be lower than one opens the possibility that both groups of depositors run simultaneously on the bank. Thus, an equilibrium will never exist where one agent runs and the other does not. Third, the bank will choose an equilibrium that allows for a run because it provides greater social welfare if inequality gets significant enough (holding all other parameters constant) while offering contracts that are still attractive to the depositors. This suggests an income inequality threshold exists where runs can occur in equilibrium. Further analysis is required to prove whether this threshold is unique.

I performed a numerical exercise to provide further insights into the theoretical results. I fixed certain parameters of the economy to study how increasing the wedge between the wealthier and poorer groups of depositors affected the probability of a run. The first result suggests that the probability of a bank run tends to one after a certain inequality threshold, holding all else equal. This analysis suggests the bank run observed in the model is a possible outcome in equilibrium, as it leads to a greater social utility when compared to cases in which a run is avoided.

A second result of the numerical exercise is that, in equilibrium, the consumption allocations increase in income inequality. As both groups are ex-ante identical in preferences and mass, the bank redistributed in terms of consumption allocation, allowing poorer households access to the larger returns of the riskier assets. Note that the participation constraint implies that the outside option is to consume their endowments. This, in turn, implies that marginal utility to consume for wealthier groups is lower as inequality increases (while the opposite holds for poorer groups) in their outside option and hence in their consumption allocations that induce their participation. This feature allows the bank to redistribute the benefits of the investment portfolio while offering attractive contracts for both groups.

The third and final result indicates that the bank's investment portfolio becomes riskier as income inequality increases. As inequality rises, the consumption allocations for all groups and types increase. When inequality is below a certain level, the portfolio is balanced between the returns of the risky and liquid assets. However, all agents engage in bank runs when inequality surpasses a certain threshold. In such cases, the bank must liquidate the risky asset at a rate lower than one, resulting in a loss. To compensate for this loss, the bank must allocate an increasing proportion of the risky asset, making the portfolio riskier overall.

In summary, the analysis demonstrates a positive relationship between income inequality and the probability of bank runs. As inequality increases, the likelihood of bank runs rises, and the investment portfolio becomes riskier. These findings shed light on how income inequality can impact financial stability and provide insights into the link between economic inequality and banking system vulnerabilities.

Literature Review. The literature has recently focused on the determinants of financial crises (see Baron et al. (2021), Gorton and Ordoñez (2020), Kirschenmann et al. (2016), Paul (2022)). Researchers have focused on a broader set of financial crises, following the definitions of Laeven and Valencia (2012), Schularick and Taylor (2012), and Reinhart and Rogoff (2009), among others. From the vast set of determinants, Kirschenmann et al. (2016) and Paul (2022) have found that income inequality has predictive power for financial crises, in a general sense, in developed countries. My contribution in this area is to document the particular correlation between income inequality (as measured by Paul (2022)) and fundamental bank runs (as defined by Baron et al. (2021)).

The literature has also examined the theoretical channel between income inequality and financial fragility. Malinen (2016) provides a brief but interesting review of this channel. For instance, Kumhof et al. (2016) finds that increasing income inequality leads to the accumulation of debt-to-income ratios, which results in a financial crisis. In summary, a relationship exists between income inequality and credit cycles in which an accumulation of credit plays a fundamental role. My contribution is establishing a channel between income inequality, and fundamental bank runs in the form of riskier investment portfolios to cover aggregate consumption that increases with inequality. These riskier investment portfolios lead to financial fragility.

This paper contributes to the income inequality and financial fragility literature (see Choi (2014) and Mitkov (2020)). More importantly, Garcia and Panetti (2022) looks into context similar to that of this paper. They investigate how wealth inequality makes financial crises more likely, finding that higher wealth inequality directly increases the incentives to run for the poor and indirectly for the rich through higher bank liquidity insurance. These incentives make self-fulfilling bank runs more likely. To reach these results, they make use of two main assumptions. First, they have multiple balance sheets that ring-fence the asset investment by wealth level, acting as universal banks. Second, they have an investment externality assumption that accounts for the contagion across wealth groups, ultimately leading to bank runs. The main difference in my contribution is that even in a model with a unique balance sheet and no investment externality, fundamental runs will happen if inequality is large enough.

Finally, this paper contributes to the literature on modeling bank runs that comes from the seminal work of Diamond and Dybvig (1983). More precisely, it extends the model presented in Allen and Gale (1998) to accommodate ex-ante identical agents with differing endowment levels to account for inequality.

This paper is divided as follows. The next section elaborates on the correlation between bank runs and income inequality. Section three presents the theoretical model and discusses some of the theoretical results. Section four presents the results of the numerical exercise, and section five concludes.

2 The Correlation Between Bank Runs and Income Inequality

In this section, I use historical data to discuss the correlation between income inequality and bank runs. First, I will describe the data used to look into this correlation. Second, I present evidence on the prevalence of bank runs in advanced economies and on how inequality behaves before such an event. Finally, I present the estimations that shed light on the correlation between bank runs and income inequality.

2.1 Data

I use two novel data sets. First, I use the data compiled by Paul (2022) on income inequality. This data set merges three long-term data sets from 1870 to 2013 for 17 countries.[1] The first long-term data set is from Òscar Jordà et al. (2016), which includes macro-financial variables for these 17 countries. The second long-term data set is from Bergeaud et al. (2016), which includes TFP and labor productivity measures. The third long-term data set is the World Inequality Database, which includes measures of income shares held by various percentiles. The novel feature found in the data set of Paul (2022) is that he includes income shares held by the upper percentiles, net of capital gains.[2]

In the second place, I use the data set on bank runs found in Baron et al. (2021). The authors collect information for 46 countries in a similar time frame as Paul (2022). More importantly, they collect bank runs narratives under a common definition to capture fundamental bank runs.[3]

The final data set includes information for 17 countries from 1880 to 2013, accounting for 2,069 country-year observations of macroeconomic, income inequality, and bank run variables.

[1] The countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

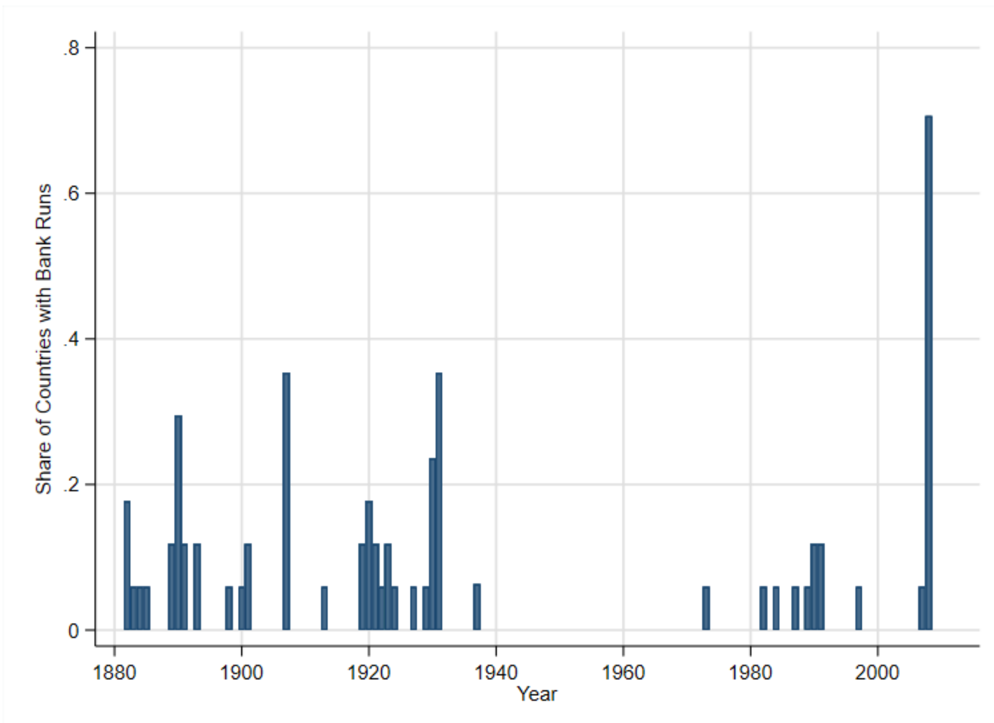
[2] The income share data is constructed using tax income data. The revenues of selling assets (i.e., stocks) are taxed depending on the tax system. The potential issue is that stock trading is typically concentrated among individuals in the upper percentile of the income distribution. This, in turn, makes capital gains available for reinvestment on the same assets, and the additional savings are not available for borrowing for other agents.

[3] Baron et al. (2021) describe bank runs as banking panics that are also bank equity crises (p. 102).

2.2 The Prevalence of Bank Runs and Inequality Trends

Figure 1 presents the prevalence of bank runs for these 17 countries between 1880 and 2013. According to Allen and Gale (2007), bank runs are nothing new and have not been restricted to emerging economies. Baron et al. (2021) suggests that there is almost no evidence of non-fundamental runs happening in this time frame. More importantly, bank runs are distributed across the time period except for the post-WWII years (i.e., 1945-1970). Using the data set, I estimate that the unconditional probability of bank runs is around 4%.

Figure 1: The Prevalence of Bank Runs

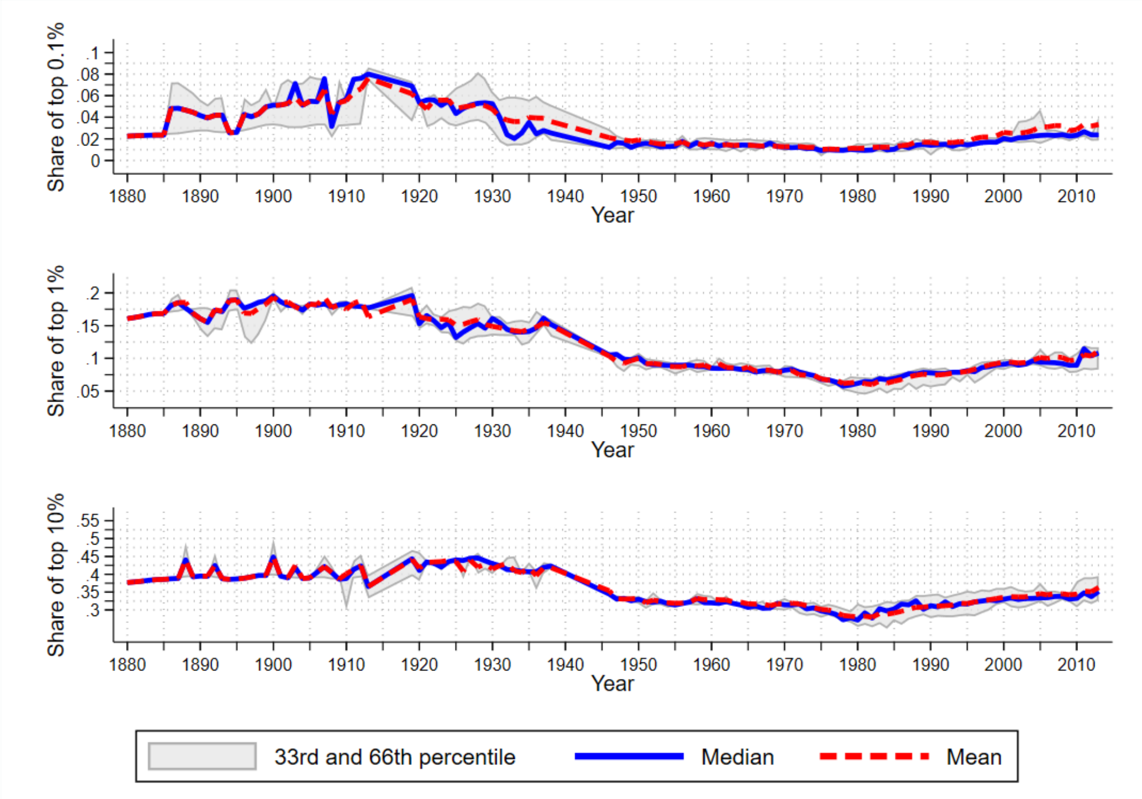


Source: Author estimation based on the data from Baron et al. (2021), Paul (2022).

Note: The sample includes 17 advanced-economy countries described in footnote 1.

The measure of inequality I use is the share of income held by the top 0.1%, 1%, and 10% of the income distribution net of capital gains found in Paul (2022). The trends of such inequality measures are presented in Figure 2. The dashed red line is the mean, while the solid blue line is the median. The borders of the grey area represent the 33rd and 66th percentiles of the inequality measures across the countries in the sample.

Figure 2: Share of Income Held by Top 0.1%, 1% and 10%.



Source: Author's estimation using Paul (2022) data.

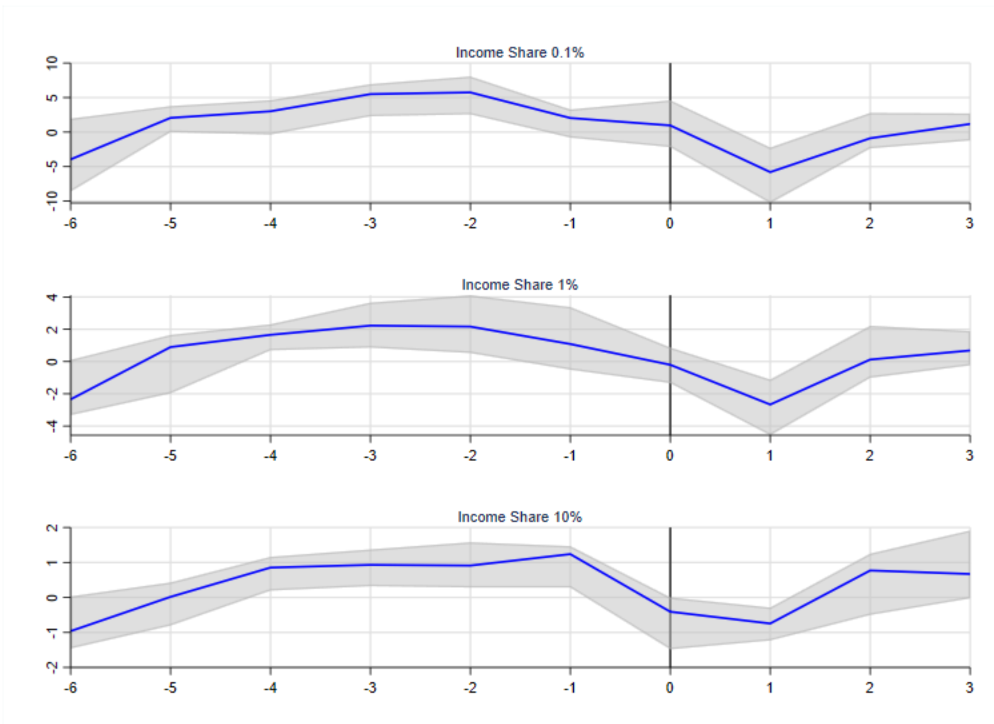
First, note that the inequality trends presented in Figure 2 have long cycles (i.e., they oscillate very slowly). For instance, they increased steadily from the late 19th century until the 1930s. Then, they decreased similarly in the post-WWII period until the 1980s, when they increased again until the end of the sample's time frame. Second, the sample median of each of the three measures is 2.8%, 12.1%, and 35.7%, for the top 0.1%, 1%, and 10%, respectively. These suggest that, at any point between 1880 and 2013, in 50% of the countries in the sample, almost 35% of the income was held by the top percentiles of the income distribution. Finally, the dispersion of such measures (i.e., the difference between the 33rd and 66th percentile) increases in periods of increasing inequality, while it collapses in periods of decreasing inequality.

These three features are by no means an exhaustive look into the inequality trends worldwide because they only apply to the countries in the sample. However, they account for the fact that inequality has increased since 1980 up to the present day (see Piketty and Saez (2003)).

2.3 Estimating the Correlation Between Bank Runs and Income Inequality

I will examine how the income shares held by the top percentiles behave around the time of a bank run. The results of this analysis are presented in Figure 3. The figure presents the median annual percentage change (in blue) and the 33rd and 66th percentile (in grey) in the years before and after a bank run.

Figure 3: Median, 33rd and 66th Percentile of Income Shares



Source: Author estimation using the data from Paul (2022) and Baron et al. (2021).

The annual percentage changes in each income share measure increase drastically in the years before the bank run, followed by a drastic fall that lasts until after the bank run, and they start recovering after that. Furthermore, note the inverted U-shape of the growth in the income inequality measures. This shape suggests that "rapid" inequality growth (i.e., changing from negative growth to positive growth from one period to another) precedes (or correlates with) an episode of bank runs. Finally, bank runs seem to reduce the income share held by the top percentiles of the income distribution (the 0.1% and the 1%), where the median reduction can go up to a 5% annual percentage change.

I want to answer the following question: Is there a (positive or negative) correlation between income inequality and bank runs? Note that I am not implying any causality because, with the data available, it is impossible to discern any causal relationship between these two phenomena. The best that can be done with the data available is to establish the type of correlation that governs the relationship between income inequality and bank runs. To do so, let the probability of a bank run be described by:

$$Pr\left(BR_{j,t}=1|\,\mathrm{Ineq}_{j,t-1},X_{j,t-1};\boldsymbol{\beta}\right)=\frac{1}{1+\exp\left(-(\alpha_j+\beta_1\Delta_h\,\mathrm{Ineq}_{j,t-1}+\beta_2\Delta_h X_{j,t-1}+\varepsilon_{j,t})\right)} \quad (1)$$

where α_j is country-specific constant, $\Delta_h\,\mathrm{Ineq}_{j,t-1}$ is the change from period $t-1-h$ to $t-1$ of either measure of inequality or a vector that includes a combination of these measures for country j, $\Delta_h X_{j,t-1}$ is the change vector of controls X from $t-1-h$ to $t-1$, and $\varepsilon_{j,t}$ is the error term. Following Paul (2022), I normalize the variables in Ineq and X by their standard deviation. The selection of h is 4, following both Paul (2022) and Gorton and Ordoñez (2020).

The results of estimating (1) are presented in Table 1. First, for the estimation of the results in columns (1), (5), and (9), I included the log change in the credit-to-GDP ratio as controls in addition to the country fixed effects. The first three columns are the estimations of each inequality measure individually.[4] The table includes the point estimation of the odds ratio, the robust standard errors in parentheses, and the marginal effect in brackets for each explanatory variable.

The results in columns (1), (5), and (9) of Table 1 suggest that a standard deviation increase in the growth of the income share held by the top percentile is correlated with a 1 to 1.2 percentage point increase in the probability of a bank run. Note that these percentage point increases in probability occur after controlling for the credit-to-GDP ratio, which has been deemed a determinant of the probability of financial crises in the literature (see Gorton and Ordoñez (2020), Paul (2022)). Remember that the sample's unconditional probability of bank runs is 4%, so a percentage point increase is a fairly significant increase in the probability of bank runs.

Taking the previous results as benchmarks, I perform additional robustness checks on estimating the correlation between income inequality and bank runs. The robustness analysis follows that in Paul (2022), and it is presented in the remaining columns of Table 1. Columns (2), (6), and (10) present the results, using as additional controls besides the 4-year change of credit-to-GDP ratio the

[4]Including any combination of income share in one estimation will produce a high correlation between explanatory variables generating biased point estimates.

Table 1: Probability of Bank Runs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_4 \log \text{Credit}_{t-1}$	0.559***	0.516**	0.648***	0.420**	0.560***	0.553**	0.650***	0.413**	0.358**	0.399*	0.504**	0.372*
	(0.146)	(0.232)	(0.231)	(0.207)	(0.129)	(0.220)	(0.212)	(0.195)	(0.163)	(0.238)	(0.231)	(0.212)
	[0.015]	[0.008]	[0.010]	[0.004]	[0.015]	[0.008]	[0.009]	[0.004]	[0.010]	[0.005]	[0.006]	[0.003]
Δ_4Income Share $0.1\%_{t-1}$	0.393***	0.578***	0.618***	0.590***								
	(0.120)	(0.181)	(0.178)	(0.201)								
	[0.010]	[0.009]	[0.009]	[0.005]								
Δ_4Income Share $1\%_{t-1}$					0.443***	0.650***	0.701***	0.517**				
					(0.155)	(0.223)	(0.204)	(0.234)				
					[0.012]	[0.009]	[0.009]	[0.005]				
Δ_4Income Share $10\%_{t-1}$									0.345*	0.585***	0.576***	0.454**
									(0.178)	(0.208)	(0.214)	(0.214)
									[0.009]	[0.008]	[0.007]	[0.004]
Set of controls		✓	✓	✓		✓	✓	✓		✓	✓	✓
GDP & Global GDP			✓	✓			✓	✓			✓	✓
Stock & House Prices				✓				✓				✓
Number of crises	28	27	27	25	34	33	33	30	26	26	26	26
Observations	808	758	758	690	952	891	891	841	812	768	768	754
Countries	15	15	15	15	17	17	17	17	17	17	17	17
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
p-value	0.337	0.484	0.377	0.861	0.380	0.643	0.606	0.620	0.283	0.531	0.494	0.731
Pseudo R^2	0.067	0.162	0.187	0.279	0.073	0.179	0.203	0.281	0.040	0.172	0.195	0.276

Source: Author's elaboration using Paul (2022) and Baron et al. (2021) data.

following (in 4-year changes): investment-to-GDP ratio, public debt-to-GDP ratio, current account-to-GDP ratio, consumer price index, long and short-term interest rates. The results suggest that even accounting for macroeconomic variables, increased inequality is correlated with an increased probability of a bank run of around one percentage point.

In columns (3), (7), and (11), the estimation additionally controls for changes in domestic and global real GDP. The results follow a similar trend as that in columns (2), (6), and (8): Their significance level increases and the point estimate of the marginal effects are smaller than those in the benchmark case. Finally, columns (4), (8), and (12) add real stock and house prices as additional controls. Note that their significance level remains relatively high for the change in the income share held by the top 0.1%. The significance level reduces for the other two measures. At the same time, the point estimates of the marginal effects are almost identical between income shares but smaller than those from the benchmark case.

In conclusion, increasing income inequality, in the form of rising income share held by the top percentiles of the income distribution, correlates with an increased probability of bank runs. This correlation suggests that an increase of one standard deviation in the growth of such shares correlates roughly with an increase of one percentage point in the probability of a bank run. These results strongly motivate studying the mechanism underpinning such correlation in a theoretical model.

3 A Model of Income Inequality and Bank Runs

In this section, I elaborate on the banking model with minimal assumptions to present the mechanism underpinning the correlation between income inequality and bank runs. First, I present the model preliminaries, the bank's constraints, and the bank's maximization problem. Next, I present the benchmark case with perfect information, followed by the different cases that arise with imperfect information. The last part of this section has a numerical exercise to represent the model features more clearly.

3.1 Preliminaries, Preferences, and Endowments

There are three periods indexed to $t = 0, 1, 2$. There is a continuum of agents with mass two composed of two groups with equal mass that differ only in their initial level of endowments. The two groups are indexed to $i = 1, 2$. Each depositor within a group is indexed to j. Without loss of generality, let the group of depositors $i = 1$ be endowed with ω_1 units of the final consumption good, and depositors in group $i = 2$ are endowed with ω_2 units, where $\omega_1 > \omega_2 > 0$. These endowment levels are common knowledge. The depositors receive the endowment at $t = 0$ and do not receive any additional endowment in $t = 1, 2$. However, they want to consume in either $t = 1$ or $t = 2$.

These depositors have liquidity preferences. That is, they are uncertain about the timing of their consumption. Furthermore, if the depositor j prefers to consume in $t = 1$, he is from type early, whereas if he prefers to consume in $t = 2$, he is from type late. These types are not common knowledge. Let the probability of being type early be $\lambda \in (0, 1)$, known to all agents. Given the equal mass of groups and by the law of large numbers, λ can be interpreted as the proportion of agents that are of type early.

The typical depositor j from group i has preferences represented by a utility function $U(c_{ti})$ that is increasing, is strictly concave, is twice continuously differentiable, and satisfies the Inada conditions. The depositor j does not know if they are from type early or late until $t = 1$. He also does not know what would happen in $t = 2$. Hence, let the expected utility of the typical depositor j in group i be described by:

$$u(c_{1i}, c_{2i}) = \mathrm{E}\left[\lambda U(c_{1i}) + (1 - \lambda) U(c_{2i})\right] \tag{2}$$

where E is the mathematical expectation. Note that to truly reveal his type, the incentive compatibility constraint for agent j in group i is given by:

$$c_{1i} \leq c_{2i} \text{ for } i = 1, 2 \tag{3}$$

Finally, to ensure that the depositor accepts the deposit contract offered by the bank, this contract has to satisfy the participation constraint in the form of

$$\mathrm{E}\left[\lambda U(c_{1i}) + (1 - \lambda) U(c_{2i})\right] \geq U(\omega_i) \text{ for } i = 1, 2 \tag{4}$$

3.2 Bank's Portfolio, Objective Function, and Constraints.

3.2.1 Banks's Portfolio

There is a bank that takes the depositors' endowments $\omega_1$and ω_2, and invests them in a portfolio composed of:

- A liquid asset (short-term) y with a constant return to scale technology that takes one unit of consumption good at t and transforms it into one unit of consumption good at $t+1$ for $t = 0, 1$. This technology can be thought of as a storage technology.

- An illiquid and risky asset x that has a constant return to scale technology that takes one unit of consumption good at $t = 0$ and transforms it into R_H units of the consumption good with probability $\pi_H \in (0,1)$ at $t = 2$ or into R_L units of the consumption good with probability $\pi_L \in (0,1)$ at $t = 2$, where $\pi_H + \pi_L = 1$ and $R_H > R_L > 1$. Therefore, there are two possible states of nature $s = H, L$. In the early liquidation of this asset, the technology takes one unit of consumption good at $t = 0$ and transforms it into $1 > r > 0$ units of the consumption good at $t = 1$.

Introducing this random asset return does not rule out bank runs that occur out of self-fulling prophecies or sunspots as a coordination mechanism. Thus, I am considering only essential bank runs (i.e., bank runs that cannot be avoided).

3.2.2 Bank's Objective Function

Given a free-entry condition, the bank will have zero profits in equilibrium. The bank provides insurance to the depositor against the preference shock while allowing the early consumer to share the higher returns of the riskier asset. Furthermore, the bank acts as a central planner whose objective is to provide a consumption allocation that maximizes a social welfare function. In this case, the bank's objective is to maximize the sum of investors' expected utility function (i.e., to maximize total surplus). This objective is characterized by

$$W\left(c_{1i}, c_{2iH}, c_{2iL}\right) = \sum_{s=H,l} \pi_s \left\{ \sum_{i=1,2} \lambda U\left(c_{1i}\right) + (1-\lambda) U\left(c_{2is}\right) \right\} \tag{5}$$

3.2.3 Bank's Constraints

Once the bank receives the endowments from the depositors at $t = 0$, it has to choose an investment portfolio $(\tilde{x}, \tilde{y})$ such that

$$\tilde{x} + \tilde{y} \leq \omega_1 + \omega_2 \tag{6}$$

This is a feasibility constraint for the bank. It suggests that the whole portfolio should be less or equal to the total endowments in the economy. I normalize ω_2 such that

$$\frac{\tilde{x}}{\omega_2} + \frac{\tilde{y}}{\omega_2} \leq \frac{\omega_1}{\omega_2} + 1 \tag{7}$$

Now, let $\omega_1 \equiv 1 + \tau$ and $\omega_2 \equiv 1 - \tau$ for a $\tau \in (0, 1)$ such that it complies with the assumption that $\omega_1 > \omega_2$ without increasing the size of the economy – that is, without making $\omega_1 + \omega_2$ greater. Then, (7) becomes

$$x + y \leq \frac{2}{1 - \tau} \equiv \omega \tag{8}$$

where $x \equiv \frac{\tilde{x}}{\omega_2}$ and $y \equiv \frac{\tilde{y}}{\omega_2}$. Note that the larger τ is, the greater inequality becomes. When $\tau \to 1$ (i.e., group one has almost all of the endowment), the inequality measure tends to infinity. Hence, I am setting τ to move freely between 0 and 1. As the bank can only purchase assets with the aggregate level of endowment in the economy, it is not ring-fencing its services to attend a specific wealth group. However, there is still a possibility for contagion among the depositors of various groups since their consumption bundles will depend on the aggregate level of endowments.

The bank has to allocate a consumption bundle for the typical agent of the group $i = 1, 2$ in the form of the triplet $c_i = (c_{1i}, c_{2iH}, c_{2iL})$. Note that the bank offers a contract in the form of $c = (c_1, c_2)$ that cannot be contingent on consumers' type (i.e., early or late), which banks cannot observe it, nor it can be contingent on the realization of state $s = H, L$.

In the first period, the bank provides a consumption c_{1i} for $i = 1, 2$ if they are of early type. The bank can allocate units of the consumption good so that the short-term asset y yields at $t = 1$. Thus, the bank's constraint at $t = 1$ is given by:

$$\lambda (c_{11} + c_{12}) \leq y \text{ for } s = H, L \tag{9}$$

This is also a feasibility constraint, suggesting that the total consumption of both groups i of agents j from the type early (i.e., the fraction λ) should not exceed the investment yield in the short-term asset y. Note that the bank will always choose $c_{11}$and c_{12} such that (9) is always satisfied. That is, it will never choose a consumption allocation in $t = 1$ such that $\lambda (c_{11} + c_{12}) > y$, because it will always default.

There is free entry, and the competition among banks leads them to maximize their consumers' expected utility, which yields zero profits in equilibrium and requires that consumers receive the entire value of the remaining assets at $t = 2$. Given that the terminal value of the assets is uncertain, the bank will find it optimal to promise a large amount that exhausts the asset's value at $t = 2$. Following Allen and Gale (2007), one can characterize the deposit contract by the parameters $c_{1i} = d_i$, where d_i is the face value of the deposit at $t = 1$. Then, (9) becomes

$$\lambda \left(d_1 + d_2\right) \leq y \tag{10}$$

If (10) holds with strict inequality, then some of the deposits can be reinvested in the short asset and consumed in $t = 2$. Thus, conditional on no run, the bank is constrained in $t = 2$ for each state s by

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s x + \left(y - \lambda\left(d_1 + d_2\right)\right) \text{ for } s = H, L \tag{11}$$

That is, the total consumption for late consumers of each group is equal to the return of the illiquid and risky asset x and whatever was stored from $t = 1$ in the form of the short-term asset y on either of the two states of nature. Note that (8) should always hold with equality to avoid any waste of endowment. Thus, we can rewrite $x = \omega - y$. Plugging this into (11), conditional on no run, the bank's constraint at $t = 2$ for each state s is

$$(1-\lambda)\left(c_{21s} + c_{22s}\right) = R_s\left(\omega - y\right) + \left(y - \lambda\left(d_1 + d_2\right)\right) \text{ for } s = H, L \tag{12}$$

The last set of constraints for the bank's problem is the incentive compatibility condition for the typical agent j of group i. This condition is given by

$$c_{2is} \geq d_i \text{ for } s = H, L \tag{13}$$

This condition suggests that consumption in $t = 2$ on either state of nature s needs to be greater or equal to the face value of the deposit at $t = 1$. This incentive compatibility condition is a necessary and sufficient condition for the agent of the group i to not run on $t = 1$.

The typical agent j will not run if and only if $c_{2is} \geq d_i$ for $i = 1, 2$ and $s = H, L$. Using (12) for agent of group $i \neq l$,

$$c_{2is} = \frac{R_s\left(\omega - y\right) + \left(y - \lambda\left(d_i + d_l\right)\right)}{(1-\lambda)} - c_{2ls} \geq d_i \Leftrightarrow$$

$$R_s\left(\omega - y\right) + \left(y - \lambda d_l\right) - (1-\lambda)c_{2ls} \geq d_i \tag{14}$$

Equation (14) is the incentive constraint for the typical agent j of the group i on each state $s = H, L$; if satisfied, late consumers will wait until $t = 2$ to consume. Then, $d_i > R_s\left(\omega - y\right) + y - \lambda d_l - (1 -$

$\lambda)c_{2ls}$ will be a necessary and sufficient condition for consumers in the group $i = 1, 2$ to run. Also, note that this incentive constraint depends on the face value of the contract of the other group d_l and the consumption allocation (c_{2ls}) for that same group at $t = 2$ on either state of nature s. Furthermore, note that there will never be a run in $s = H$ without a run in $s = L$ because $R_H > R_L$. This implies that the bank will not choose d_i so large that $d_i > R_H(\omega - y) + (y - \lambda d_l) - (1 - \lambda) c_{2ls}$. Therefore, I focus on cases where a run could happen in $s = L$ if it occurs. One only needs to be concerned about the incentive constraint for state L. That is, there will be no run if and only if

$$R_L(\omega - y) + (y - \lambda d_l) - (1 - \lambda)c_{2lL} \geq d_i \text{ for } i, l = 1, 2 \text{ and } i \neq l \tag{15}$$

If there is a run, the bank must liquidate all the long-run assets in $t = 1$ at the fire-sale rate. That is, if group i runs, the deposit contract d_i becomes

$$d_i = rx + y - \lambda d_l \text{ for } i, l = 1, 2, i \neq l \tag{16}$$

Consequently, the bank's problem is given by

$$\max_{d_1, d_2, y, \{c_{2is}\}_{i=1,2, s=H,L}} \sum_{s=L,H} \pi_s \{\lambda [U(d_1) + U(d_2)] + (1 - \lambda) [U(c_{21s}) + U(c_{22s})]\} \tag{17}$$

subject to

$$\lambda(d_1 + d_2) \leq y \tag{18}$$

$$(1 - \lambda)(c_{21s} + c_{22s}) \leq R_s(\omega - y) + y - \lambda(d_1 + d_2) \text{ for } s = H, L \tag{19}$$

$$c_{2iL} \geq d_i \text{ for } i = 1, 2, \tag{20}$$

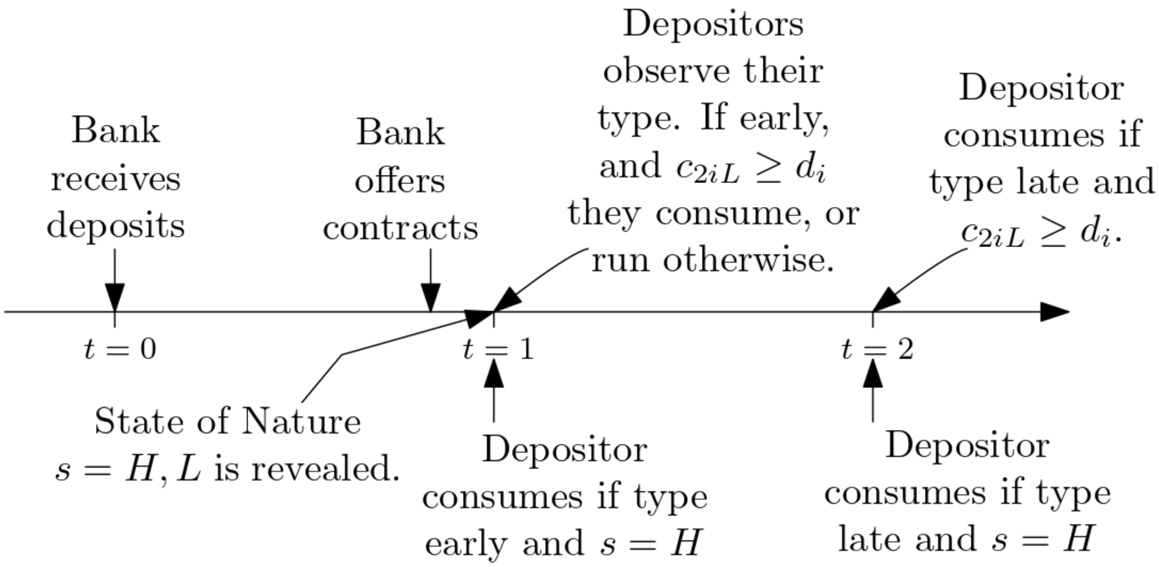
The direction of the inequality in (20) depends on whether a run occurs.

The timing of the problem is presented in Figure 4. The bank receives the deposits at $t = 0$ and offers the deposit contracts by the end of that period. Most of the action occurs at $t = 1$. In this period, the depositor's type and the state of nature are revealed. Then, if the state is H, the depositors will withdraw at their respective periods, and there will be no run. Otherwise, if the state is L and the IC constraints hold, the depositors will withdraw at their respective periods, and there will be no run. However, if the IC constraint is violated, a run will occur.

3.3 Benchmark Case: Perfect Information

First, I will develop the model with perfect information as a benchmark for the following results. In an economy with perfect information, the bank knows exactly the state of nature that will occur.

Figure 4: The timing of the porblem



Hence, the bank will solve two separate problems that are characterized by the following:

$$\max_{d_1,d_2,c_{21s},c_{22s},y} \sum_{i=1,2} \lambda U\left(d_i\right) + (1-\lambda)\, U\left(c_{2is}\right) \tag{21}$$

subject to (18) and (19). Note that with perfect information, the bank will never offer a contract such that any group i will run in either state. Thus, (20) will hold with strict inequality.

From (19), let

$$\widetilde{c}_{22s}(d_1, d_2, c_{21s}, y) = \frac{R_s(\omega - y) + y - \lambda(d_1 + d_2) - (1-\lambda)c_{21s}}{1-\lambda} \tag{22}$$

Then, the problem in (21) becomes

$$\max_{d_1,d_2,c_{21s},y} \lambda \left[U\left(d_1\right) + U\left(d_2\right)\right] + (1-\lambda)\left[U\left(c_{21s}\right) + U\left(\widetilde{c}_{22s}(d_1, d_2, c_{21s}, y)\right)\right] \tag{23}$$

The first-order conditions, alongside the fact that it will never be optimal to have a short-term investment (y) from $t = 1$ into $t = 2$ (see Allen and Gale (2007)), results in the set of Euler equations for states $s = H, L$ characterized by:

$$U'\left(d_i\right) = R_s U'\left(\frac{R_s(\omega - 2\lambda d_i)}{2(1-\lambda)}\right) \text{ for } i = 1, 2 \tag{24}$$

First, for each group i, (24) implies that $c_{2is} > d_i$ since $R_s > 1$. Thus, the incentive compatibility constraint is satisfied. Also, note that, in the case of perfect information, the deposit contracts in

$t = 1$ for both groups of agents are equal (i.e., $d_1 = d_2$). This is because the marginal utility of consumption is equalized at any given consumption allocation between agents in $t = 1$ and $t = 2$.

Third, the Euler equation intuitively suggests that the marginal cost of the deposit contract d_i should be equal to the marginal benefit of consumption in $t = 2$ at any state s. Given the assumptions on $U(\cdot)$, the deposit contract at $t = 1$ for agent i is given by:

$$d_i = \frac{U'^{-1}(R_s)R_s\omega}{2(1-\lambda(1-U'^{-1}(R_s)R_s))} \text{ for } i = 1, 2 \tag{25}$$

$$c_{2is} = \frac{R_s\omega}{2(1-\lambda(1-U'^{-1}(R_s)R_s))} \text{ for } i = 1, 2, s = H, L \tag{26}$$

The deposit contract increases with inequality, as captured by $\omega = \frac{2}{1-\tau}$. This, in turn, implies that the consumption allocation depends on the aggregate level of endowments, suggesting a cross-subsidy between the wealth groups to attain a higher consumption bundle than they would get if they only consumed their endowments. As τ becomes greater, the wealthier group holds the more significant proportion of the overall endowment. However, with the deposit contract equal across both groups, the bank compensates for the consumption of the less wealthy group by sharing the returns on the risky asset accordingly. Since $R_s > 1$ for any state, both groups of depositors are better off than consuming only their endowment.

3.4 Bank's Possible Cases with Imperfect Information

Now, the bank cannot know which state of nature occurs. Then, the problem in (17)-(20) will lead to various cases due to how the incentive constraint binds (or not) for each agent of the group $i = 1, 2$. The cases are described in Table 2.

Table 2: Possible Cases Given How the Incentive Constraint Binds

Case	1	2	3	4	5	6	7	8	9
c_{21L}	$> d_1$	$> d_1$	$> d_1$	$= d_1$	$= d_1$	$= d_1$	$< d_1$	$< d_1$	$< d_1$
c_{22L}	$> d_2$	$= d_2$	$< d_2$	$> d_2$	$= d_2$	$< d_2$	$> d_2$	$= d_2$	$< d_2$

Note that in case of a run, the bank has to liquidate all long-run assets in $t = 1$ at a fire sale rate $r \leq 1$. Hence, if the group i runs on the bank, the consumption in $t = 1$, which is the deposit contract d_i, becomes

$$d_i = r(\omega - y) + y - \lambda d_l \text{ for } i, l = 1, 2, i \neq l \tag{27}$$

If all the long-run asset is liquidated, it is possible that $c_{2lL} = 0 < d_l$. Given the assumptions on the preferences of the depositors, $d_l > 0$, which in turn prompts depositors in the group l to run on the bank. Then,

$$d_l = r(\omega - y) + y - \lambda d_i \text{ for } l, i = 1, 2, l \neq l \tag{28}$$

Plugging (28) into (27),

$$d_i = \frac{r(\omega - y) + y}{(1 + \lambda)} \text{ for } i = 1, 2 \tag{29}$$

From the previous discussion, bank runs under this setting are contagious, leading to ruling out cases where one agent runs, and the other does not. Thus, the remaining cases are in Table 3.

Table 3: Remaining Possible Cases

Cases	1	2	3	4	5
c_{21L}	$> d_1$	$> d_1$	$= d_1$	$= d_1$	$< d_1$
c_{22L}	$> d_2$	$= d_2$	$> d_2$	$= d_2$	$< d_2$

I will discuss cases 1, 2, 4, and 5 in what follows. Case 3 is the same as case 2, with the subscripts exchanged. Finally, I assume the participation constraint holds for all the cases considered here.

Let $W_k(d_1^k, d_2^k, c_{21L}^k, c_{22L}^k, c_{21H}^k, c_{22H}^k)$ be the social utility function valued at the optimal deposit contracts for the case $k = 1, 2, 3, 4, 5$. This function will be an increasing function of ω, given that all consumption bundles are increasing functions of this parameter. The bank will select the corresponding case, given a set of parameters, with the following decision rule:

$$W_k(\cdot) \max W_{-k}(\cdot) \tag{30}$$

where $-k$ are all other cases but the $k^t h$ case. By allowing the indifference to be solved in favor of the case with greater utility, I focus on the "best" equilibrium selection scenario.

3.4.1 Case 1: Incentive Constraint is Never Binding for Both Agents

In this case, the bank's maximization problem is described by equations (17)-(19). The incentive constraint in (20) holds with strict inequality. From (19), let

$$\tilde{c}_{22s}(d_1, d_2, y, c_{21s}) \equiv c_{22s} = \frac{R_s(\omega - y) + y - \lambda(d_1 + d_2) - (1 - \lambda)c_{21s}}{1 - \lambda} \text{ for } s = H, L \tag{31}$$

Then, the problem becomes

$$\max_{d_1,d_2,c_{21H},c_{21L},y} \sum_{s=H,L} \pi_s \left\{\lambda\left[U\left(d_1\right)+U\left(d_1\right)\right]+(1-\lambda)\left[U\left(c_{21s}\right)+U\left(\tilde{c}_{22s}(d_1,d_2,c_{21s})\right)\right]\right\} \tag{32}$$

subject to

$$y=\lambda(d_1+d_2) \tag{33}$$

As in the benchmark case, $d_1 = d_2$ from the first-order conditions. As such, the Euler equation for this case becomes

$$U'\left(d_i\right)=\pi_H R_H U'\left(\frac{R_H(\omega-2\lambda d_i)}{2(1-\lambda)}\right)+\pi_L R_L U'\left(\frac{R_L(\omega-2\lambda d_i)}{2(1-\lambda)}\right) \text{ for } i=1,2 \tag{34}$$

Note that this Euler equation has a similar intuition as that of the benchmark case, with the main difference being the right-hand side. In this case, the right-hand side is the expectation of the marginal benefit of future consumption. Note that under the assumption, $R_H > R_L > 1$, thus implying that in expectation, the consumption in $t = 2$ is greater than in $t = 1$. Thus, the incentive constraint is satisfied. Furthermore, consumption allocations for both groups will increase with the level of inequality, as in the benchmark case.

3.4.2 Case 2: Incentive Constraint is Never Binding for $i = 1$ and Just Satisfied for $i = 2$.

In this case and case 3, neither agent will run on the bank. The main difference is that the incentive constraint binds for the depositors of the group $i = 2$ while it does not bind for those in the group $i = 1$, while the opposite will happen in case 3. Now, for the state H, equation (19) can be characterized by:

$$\tilde{c}_{22H}(d_1,d_2,c_{21H},y)\equiv c_{22H}=\frac{R_H(\omega-y)+y-\lambda(d_1+d_2)-(1-\lambda)c_{21H}}{1-\lambda} \tag{35}$$

Moreover, given that $c_{22L} = d_2$ and using (19) for state L,

$$\tilde{d}_2(d_1,c_{21L},y)\equiv d_2=R_L(\omega-y)+y-\lambda d_1-(1-\lambda)c_{21L} \tag{36}$$

Now, plugging (36) into (35),

$$\tilde{c}_{22H}(d_1,c_{21L},c_{21H},y)=\frac{(R_H-\lambda R_L)(\omega-y)+(1-\lambda)y-\lambda(1-\lambda)(d_1-c_{21L})-(1-\lambda)c_{21H}}{1-\lambda} \tag{37}$$

Similarly, the feasibility constraint for $t = 1$ becomes:

$$y(1-\lambda(1-R_L))=\lambda(1-\lambda)(d_1-c_{21L})+\lambda R_L\omega \tag{38}$$

Then, the bank's problem becomes:

$$\max_{d_1, c_{21H}, c_{c21L}, y} \lambda U(d_1) + (\lambda + (1-\lambda)\pi_L) U\left(\tilde{d}_2(d_1, c_{21L}, y)\right)$$
$$+ (1-\lambda)\left\{\pi_H\left[U(c_{21H}) + U(\tilde{c}_{22H}(d_1, d_2, c_{21s}))\right] + \pi_L U(c_{21L})\right\} \quad (39)$$

subject to (38). First, note that for state H, given a level of short-term asset y, agents will split the consumption evenly. That is,

$$c_{21H} = \frac{R_H(\omega - y)}{2(1-\lambda)} = c_{22H} \quad (40)$$

Second, with the first-order conditions with respect to d_1 and y, I have the following expression:

$$U'(d_1) = \left[\pi_H R_H U'\left(\frac{R_H(\omega - y)}{2(1-\lambda)}\right) + \pi_L R_L U'\left(\tilde{d}_2(d_1, y, c_{21L})\right)\right]$$
$$+ \frac{\lambda R_L}{1-\lambda}\left[U'\left(\tilde{d}_2(d_1, y, c_{21L})\right) - U'(d_1)\right] \quad (41)$$

This last expression would resemble the Euler equation in the previous case if it were not for the additional term on the right-hand side. This additional term could be understood as some subsidy or penalty, depending on whether $\tilde{d}_2(\cdot)$ is greater than d_1, on the marginal benefit of the wealthier group 1.

Finally, from the first-order conditions concerning d_1 and c_{21L}, the size of the additional term in (41) is given by

$$\frac{\lambda R_L}{1-\lambda}\left[U'\left(\tilde{d}_2(d_1, y, c_{21L})\right) - U'(d_1)\right] = R_L \pi_L \left[U'(c_{21L}) - U'\left(\tilde{d}_2(d_1, y, c_{21L}\right)\right] \quad (42)$$

which is the value of the difference in the marginal utility of the consumption of the wealthy group 1 and the consumption of the less wealthy group 2. If I plug (42) into (41),

$$U'(d_1) = \left[\pi_H R_H U'\left(\frac{R_H(\omega - y)}{2(1-\lambda)}\right) + \pi_L R_L U'(c_{21L})\right] \quad (43)$$

This is the typical Euler equation for the depositors in the group 1, whose intuition is similar to the one described in the benchmark case. The marginal cost of consuming today equals the marginal benefit of future consumption. More importantly, given (38), the selection of y depends on both d_1 and c_{21L}. Thus, $\tilde{d}_2(\cdot)$ depends exclusively on d_1 and c_{21L}. Specifically, given an allocation of d_1 and c_{21L}, I can characterize d_2 as

$$d_2 = \frac{R_L \omega}{1 - \lambda(1 - R_L)} + \frac{(1 - R_L)\lambda(1-\lambda)}{1 - \lambda(1 - R_L)}(d_1 - c_{21L}) \quad (44)$$

The coefficient accompanying $d_1 - c_{21L}$ is negative. Given the assumption that $c_{21L} > d_1$, the difference is also negative. Thus, the second term is positive. This suggests a trade-off for the bank when allocating d_2. On one hand, increasing inequality increases the face value of the contract at $t = 1$, so depositors in the group $i = 2$ do not run the bank, holding constant the contract offered to depositors in group $i = 1$. On the other hand, holding the inequality level constant, a larger face value deposit means a smaller difference between the face value of the deposit in $t = 1$ and the consumption allocation in $t = 1$ for depositors of the group $i = 1$.

3.4.3 Case 4: Incentive Constraint is Just Binding for Both Groups

This case is special in that the two groups have the incentive constraint just satisfied – that is, $c_{21L} = d_1$ and $c_{22L} = d_2$. Given these two conditions, and using (19) for state L,

$$c_{22L} = R_L(\omega - y) + y - c_{21L} \tag{45}$$

On the other hand, for state H, equation (19) becomes

$$c_{22H} = \frac{(R_H - \lambda R_L)(\omega - y) + (1-\lambda)y - (1-\lambda)c_{21H}}{1-\lambda} \tag{46}$$

Furthermore, with (45), I can rewrite (18) as

$$y = \frac{\lambda R_L \omega}{1 - \lambda(1 - R_L)} \tag{47}$$

Note that with (47), I have determined the equilibrium investment for this case. This investment is increasing in inequality because it is increasing in ω. Now, plugging this last expression into (45) and (46),

$$c_{22H} = \frac{R_H \omega}{1 - \lambda(1 - R_L)} - c_{21H} \tag{48}$$

$$c_{22L} = \frac{R_L \omega}{1 - \lambda(1 - R_L)} - c_{21L} \tag{49}$$

Consequently, the bank maximization problem becomes

$$\max_{c_{21L}, c_{21H}} (\lambda + (1-\lambda)\pi_L) \left[U(c_{21L}) + U\left(\frac{R_L \omega}{1 - \lambda(1 - R_L)} - c_{21L} \right) \right] + (1-\lambda)\pi_H \left[U(c_{21H}) + U\left(\frac{R_H \omega}{1 - \lambda(1 - R_L)} - c_{21H} \right) \right] \tag{50}$$

The first-order conditions with respect to c_{21L} and c_{21H} suggest that

$$c_{21L} = \frac{R_L \omega}{2(1 - \lambda(1 - R_L))} = c_{22L} = d_1 = d_2 \tag{51}$$

$$c_{21H} = \frac{R_H \omega}{2(1 - \lambda(1 - R_L))} = c_{22H} \tag{52}$$

With equations (51) and (52), I can define the indirect utility function. That is,

$$W(R_L, R_H, \omega, \lambda, \pi_H, \pi_L) = (\lambda + (1-\lambda)\pi_L) 2U\left(\frac{R_L\omega}{2(1-\lambda(1-R_L))}\right) + (1-\lambda)\pi_H 2U\left(\frac{R_H\omega}{2(1-\lambda(1-R_L))}\right) \quad (53)$$

Note that this function is increasing in inequality because the consumption in either state increases in ω. This occurs because of the equal split of the consumption by the groups of depositors, and the wealthier depositors are increasing the poorer depositors' consumption by depositing the endowments in the same financial institution.

3.4.4 Case 5: Incentive Constraint is Violated for Both Types of Depositors.

In this case, remember that the relevant incentive constraint is for the state $s = L$. Then, it is still true that the depositors will not run on the bank at state $s = H$, which implies that (19) for this state can be rewritten as:

$$\tilde{c}_{22H} \equiv c_{22H} = \frac{R_H(\omega - y) + y - \lambda(d_1 + d_2) - (1-\lambda)c_{21H}}{1-\lambda} \quad (54)$$

However, the deposit contract in the case of state $s = L$ materializing has the form described in (29)

$$d_i = \frac{r(\omega - y) + y}{(1+\lambda)} = c_{2iL} \text{ for } i = 1, 2 \quad (55)$$

Thus, using (54) and (55), the bank's maximization problem becomes

$$\max_{d_1, d_2, c_{21H}, y} \pi_H \left\{\lambda \left[U(d_1) + U(d_2)\right] + (1-\lambda)\left[U(c_{21H}) + U(\tilde{c}_{22H}(d_1, d_2, c_{21H}, y))\right]\right\} + 2\pi_L U\left(\frac{r(\omega - y) + y}{(1+\lambda)}\right) \quad (56)$$

subject to (18) holding with equality. Given the first-order conditions for d_1 and d_2, $d_1 = d_2$ for the state $s = H$. Along with (18), $y = 2\lambda d_1$. Furthermore, the first-order condition for c_{21H} suggests that

$$c_{21H} = \frac{R_H(\omega - 2\lambda d_1)}{2(1-\lambda)} \quad (57)$$

Finally, with the first-order conditions for d_1 and y, I have the following Euler equation:

$$U'(d_1) = \frac{1}{\pi_H}\left[R_H \pi_H U'\left(\frac{R_H(\omega - 2\lambda d_1)}{2(1-\lambda}\right) + r\pi_L U'\left(\frac{r\omega + (1-r)2\lambda d_1)}{2}\right)\right] - \frac{\pi_L}{\pi_H} U'\left(\frac{r\omega + (1-r)2\lambda d_1}{(1+\lambda)}\right) \quad (58)$$

The intuition behind equation (58) is as follows. The marginal cost of consumption in $t = 1$ is equal to the expected marginal benefit of consumption in $t = 2$ weighted by the inverse of the probability of the realization of state H (i.e., $\frac{1}{\pi_H}$), plus the odds of the L state times the marginal utility of consumption if the depositors were to run. Because the marginal utility is always positive by assumption, and the odds are also greater than 0, the second term on the right-hand side decreases the marginal benefit of future consumption.

3.4.5 Case selection

To simplify the case selection explanation, take only the polar cases: case 1 (no run) and case 5 (run). According to the decision rule in (30), the bank will select case 1 above case 5 whenever

$$W_1(d_1^1, d_2^1, c_{21L}^1, c_{22L}^1, c_{21H}^1, c_{22H}^1) \geq W_5(d_1^5, d_2^5, c_{21L}^5, c_{22L}^5, c_{21H}^5, c_{22H}^5) \tag{59}$$

Note that in all cases, $d_1^k = d_2^k$ and $c_{21s}^k = c_{22s}^k$ for $k = 1, 5$ and $s = H, L$. Then, (59) becomes

$$\lambda U(d_1^1) + (1-\lambda)\left(\pi_H U(c_{21H}^1) + \pi_L U(c_{21L}^1)\right) \geq \pi_H \left(\lambda U(d_1^5) + (1-\lambda) U(c_{21H}^5)\right) + \pi_L U(c_{21L}^5) \tag{60}$$

Now, the consumption allocations depends on $\omega = \frac{2}{1-\tau}$, hence (60) also depends on τ. Thus, $\hat{\tau} \in (0,1)$ should exist as a function of the economy's parameters that solves (60). So far, without assuming any functional form, this $\hat{\tau}$ is still an implicit function. To find this τ, I will employ the numerical exercise described in the next section.

3.5 Numerical Exercise

In this subsection, I present the results of a numerical exercise to showcase some of the properties of the model. The main goal is to look at how a change in τ, which implies changing the level of inequality, affects the various consumption allocations, the investment, the welfare function, and, more importantly, the probability of a bank run.

$$U(c) = \frac{(c+\psi)^{1-\gamma} - \psi^{1-\gamma}}{1-\gamma} \tag{61}$$

where γ is the parameter that rules the level of risk aversion of the typical depositor, and ψ is a parameter that satisfies a modified Inada condition. This parameter can be interpreted as a minimum consumption level.[5]

[5] Garcia and Panetti (2022) use this utility function. Note that this utility is twice continuously differentiable, increasing, and concave. Additionally, it satisfies $U(0) = 0$ and $\lim_{c \to 0} U'(c) = F < \infty$. These last are the modified Inada conditions.

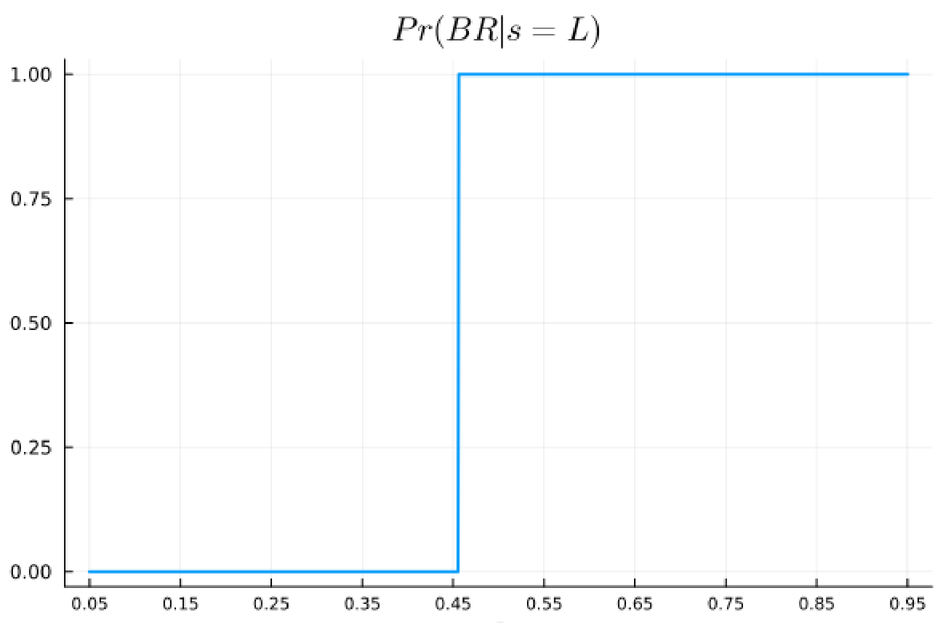
Table 4: Set of parameters for numerical exercise

Parameter	λ	τ	R_H	R_L	γ	ψ	r	π_H
Value	0.15	[0.05,0.95]	2.0	1.5	3.0	1.0	0.8	0.85

The parameters used in the numerical exercise are presented in Table 4. The parameters R_H and R_L imply that the risky asset pays two units of consumption goods per unit invested when it matures in the state H or 1.5 units in the state L. The parameter r implies that the recovery rate of the risky asset when liquidated early is about 80% the original investment value. I used $\gamma = 3$ because it is a standard risk aversion parameter. I set the parameter ψ at 1.0. Finally, I set the parameter λ, the share of type early depositors, at 15%.

For the probability of the state H, I used a value of 85%. These, in turn, suggest that the probability of a low state is 15%. Lastly, I set τ to move freely between 0.05 to 0.95. Remember that a greater value of τ implies a bigger wedge between wealthier and poorer groups.

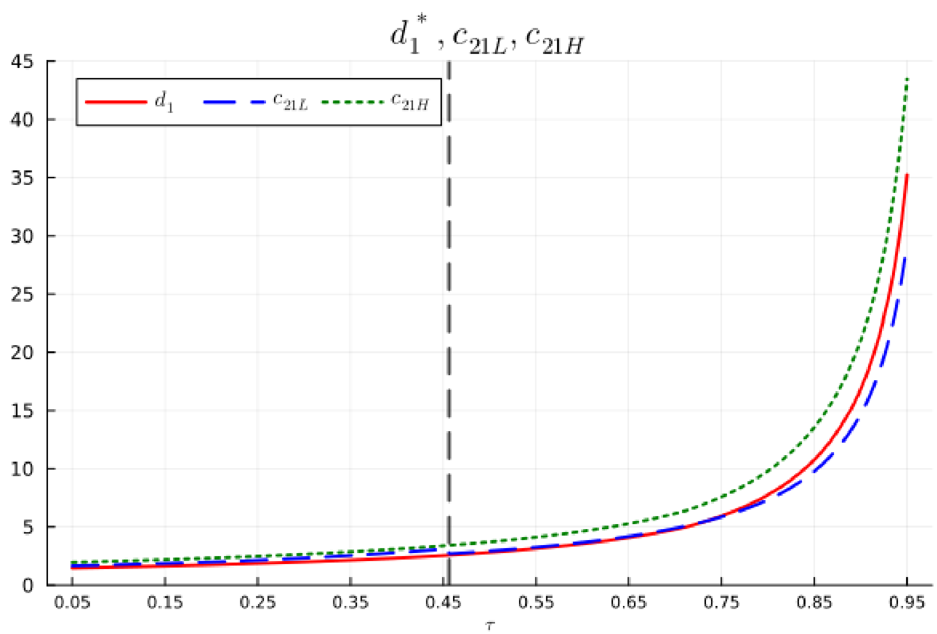
Figure 5: Conditional Probability of Bank Run



The first result is that the probability of a bank run is increasing in τ. This result is presented in Figure 5. First, note that given the structure of the problem, runs can only happen in the low state. Thus, the probability of a bank run, conditional on being in the state L, is one once it reaches a sufficiently high τ (i.e., 0.46 in this case). The unconditional probability is given by $Pr(BR|s = L) \times Pr(s = L) = \pi_L = 0.15$. The jump in the probability of a bank run is due to the discrete nature of the actions of either agent (i.e., to run or not to run). Second, note that the run characterized here is allowed in equilibrium because the bank finds it will render a larger utility than a scenario in which the incentive compatibility constraints were holding (binding or not). This first result confirms a correlation between income inequality and bank runs in the data. As inequality rises (i.e., larger τ), the probability of the bank run increases.

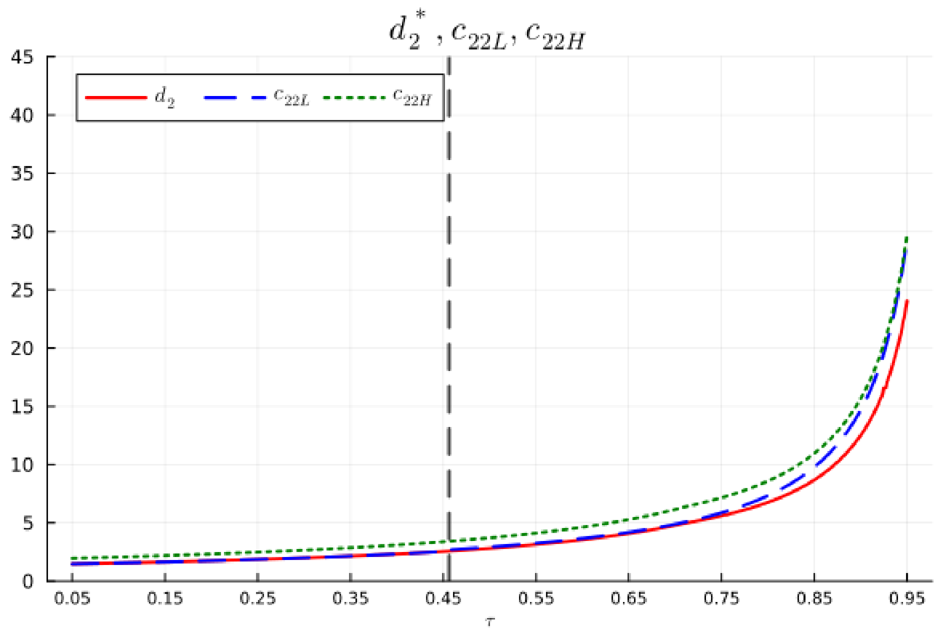
The second result is that deposit contracts are increasing with τ. This result is presented in Figures 6 and 7. The consumption allocations were identical in equilibrium for both groups because they depended on the aggregate level of the endowment, and both groups were equal in size. Thus the bank was "neutral" on how the endowment distribution was ex-ante. With the unique balance sheet,

Figure 6: Equilibrium Consumption Allocations for group $i = 1$



the bank redistributed in terms of consumption allocations, allowing the poorer households access to the larger returns of the riskier asset. More importantly, note that the participation constraint is increasing (decreasing) in τ for the wealthier (poorer) depositors. That is, the marginal utility to consume for the wealthier groups is lower as inequality increases, while it is larger for poorer groups. This allows the bank to redistribute benefits equally while offering attractive contracts for both groups.

Figure 7: Equilibrium Consumption Allocations for group $i = 2$

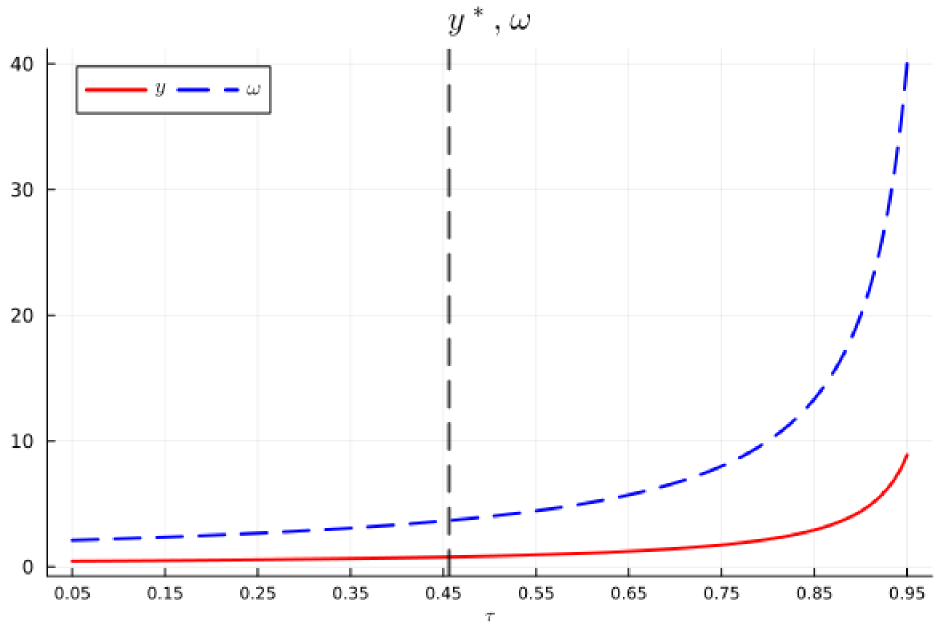


The third and final result is that the investment portfolio is riskier with increasing inequality. This result is shown in Figure 8. The intuition behind this result is as follows: As inequality increases with τ, the consumption allocations increase for all groups and types. For $\tau < 0.46$, the portfolio tends to be balanced between the returns of the risky asset in either state and the storage capacity of the liquid asset. This balanced portfolio happens to keep up with the increasing consumption allocations. For $\tau > 0.46$, the agents are all running. In this case, the liquid asset that was invested early will return its usual 1 unit, but now, the bank has to liquidate the risky asset at a rate of $r < 1$. Hence, the bank requires an increasing proportion of the risky asset to compensate for the loss of the fire-sale rate. This result suggests that the portfolio has to become increasingly riskier (i.e., a larger proportion of the riskier asset) as inequality increases.

4 Conclusion

In conclusion, this study contributes to the literature on the correlation between income inequality and financial crisis, particularly its association with bank runs. The analysis reveals a positive correlation between income inequality and the likelihood of bank runs. This finding relates to the importance of understanding the determinants of bank runs, as they are recognized as triggers for financial crises. Policymakers can benefit from this knowledge by designing effective strategies to

Figure 8: Investment Portfolio



mitigate the adverse impact of financial crises occasioned by bank runs.

Furthermore, the study examined the mechanism underlying the connection between income inequality and bank runs. I constructed a model with a set of minimal assumptions that suggest: i) a higher level of income inequality leads to a greater probability of bank runs, ii) equilibrium consumption allocations are increasing in inequality, suggesting a redistribution by the bank without compromising the attractiveness of the deposit contracts, and iii) a riskier investment portfolio is required to cover for the increasing consumption allocations, which generate more financial fragility.

Altogether, these results demonstrate that the bank's liquidity transformation role and a possible redistribution role can impact the system's soundness. By comprehending this underlying mechanism, policymakers can implement targeted policies to address the impact of income inequality on bank runs, thereby enhancing financial stability and reducing the likelihood and severity of financial crises. In conclusion, this research provides valuable insights into the relationships between income inequality, bank runs, and financial crises.

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