Indirect tax evasion, shadow economy, and the Laffer curve: A theoretical approach*

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Abstract

This paper provides new theoretical insights into the causes and consequences of indirect tax evasion. I propose a decision-making framework that contemplates biased perceptions of apprehension probabilities, which are affected by the environment where the agents operate. This micro-founded formulation allows for the analysis of how taxation affects tax evasion (and vice versa) in the aggregate, emphasizing the existing relationships between the relative size of the shadow economy, tax rates, and government revenue. It is shown that a traditional Laffer curve (inversely U-shaped and with a unique maximum) can only exist under certain conditions. The maximum government revenue attainable turns out to be, in any case, lower than in the absence of tax evasion. Nevertheless, evasion control policies are proven to be always effective in increasing government revenue.

Keywords: Indirect tax evasion; Law and Economics; Biased perceptions.

JEL Classification: D80, H26, K42.

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1 Introduction

Since Becker's (1968) seminal work, economists have systematically studied criminal behavior using his proposed methodology. Briefly, potential offenders are considered rational agents who operate in a context of risk or uncertainty regarding the possible outcomes of their offenses (they *might* be detected, apprehended, and punished), choosing the delinquent activity level that maximizes their expected utility.

Tax evasion models first appeared in the 1970s after Allingham and Sandmo's (1972) pioneering paper, which was the first application of Becker's theory to this specific type of felony. Since then, several models for tax evasion have been developed and numerous empirical researches have been conducted.

This paper presents an indirect tax evasion model with a decision-making framework that accounts for some of the empirical observations often overlooked by the literature.[1] The manuscript is structured as follows: Section 2 presents the issues addressed in this paper, Section 3 develops the model at a single-agent level, Section 4 analyzes tax evasion in the aggregate, Section 5 simulates an economy with heterogeneous agents to evaluate the effects of different policy tools, and Section 6 provides a conclusion.

2 Considerations

Compelling empirical evidence suggests that potential tax evaders usually overestimate the objectively low probability of being caught, thus resulting in lower levels of tax evasion than predicted by traditional models (Alm, 2019; Andreoni et al., 1998). One possible explanation for this phenomenon is the presence of biases and heuristics that influence perceived probability distributions over outcomes.[2]

Furthermore, decisions regarding tax evasion are significantly influenced by the social environment (shaped by the taxpayers and the tax authority) in which they are made. This environment does not only affect the perceived probabilities for each outcome (Bergman & Nevarez, 2005; Cooter et al., 2008; Scholz & Pinney, 1995; Sheffrin & Triest, 1992) but also the preferences[3] over them (Alm & McClellan, 2012; Bergman & Nevarez, 2006; Frey & Feld, 2002; Scholz & Pinney, 1995; Torgler et al., 2008).

In recent years, mainly for the case of income tax evasion, new models have been formulated to address the mentioned issues. Some of these models focus on the interactions between agents (degl'Innocenti & Rablen, 2017; Di Gioacchino & Fichera, 2020, 2022; Fortin et al., 2007; Traxler, 2010), while others place

[1]The Annex contains two lists with the nomenclature for each type of variable (individual and aggregate) and also presents the notation I use.

[2]See Kahneman et al. (1982) or Thaler and Sunstein (2008) for a more in-depth discussion of those biases and heuristics.

[3]For instance, the utility level for a certain amount of successful tax evasion may depend on factors such as how taxpayers are treated by the tax authority or how much trust the government has earned.

emphasis on the impact of tax audits (Levaggi & Menoncin, 2016; Ma et al., 2021). Additionally, some models analyze tax evasion decisions using a non-expected utility framework (Bernasconi, 1998; Bernasconi & Zanardi, 2004; Frey & Torgler, 2007; Hokamp & Pickhardt, 2010; Yaniv, 1999). In contrast, the modern indirect tax evasion theory (Arias, 2011; Besfamille et al., 2009, 2013; Buccella et al., 2024; Fanti & Buccella, 2021; Goerke, 2017; Goerke & Runkel, 2011) is still mainly concerned with the same issues as in its early stages (Marrelli & Martina, 1988; Marrelli, 1984; Virmani, 1989). This is, the study of the relationships between tax evasion, output, and production efficiency under a determinate market structure.

It may be said that this split in new theoretical modeling is because the empirical findings mentioned are only valid for individuals and not for firms. However, firm decisions are taken by individuals operating within an organizational structure (Coase, 1937, 1988). Therefore, it might not be appropriate to claim that individuals are perfectly rational and well-informed when making indirect tax evasion decisions in a company but not when evading income tax on their personal earnings.[4]

Even if it is accepted that the aforementioned empirical insights ought to be incorporated into indirect tax evasion models, a challenging issue to overcome is the incongruity that arises between tax evasion decisions and production decisions when *irrational* formation of probability perceptions is taken into account. It would be inconsistent to accept that the former are made under total uncertainty (leading to the formation of biased probabilities of detection on which an agent's behavior relies) at the same time that the latter take place in a context of perfect and complete information (as assumed by traditional market structures).

One potential solution is to separate tax evasion decisions from production decisions and focus solely on the former. Although this would imply the absence of a market structure, it does not necessarily mean that revenue is to be taken as given. It could be treated as an endogenous variable, influenced by factors such as tax rates. Furthermore, this approach would allow for a broader aggregate analysis, as it overcomes certain limitations imposed by each market structure.[5]

Certainly, if probability of detection or concealment costs depend on firm size, the approach would contradict the well-known *separability* conclusions of indirect tax evasion theory (Arias, 2011; Yaniv, 1995). However, it is worth noting that tax evasion decisions and production decisions are not made simultaneously, as it is usually[6] assumed, but rather at different times. Therefore, it might not be incorrect to focus on the second stage of the problem, this is, on how much revenue is understated, rather than how it is generated (even though this may affect tax evasion decisions).

[4]As an example, Busenitz and Barney (1997) show that both managers of large corporations and sole entrepreneurs, yet to a different extent, are prone to misperceive probabilities due to behavioral biases when facing uncertainty.

[5]As an example, it would take into account the existence of heterogeneous agents, which is not possible in a market of perfect competition.

[6]For a two-stage model, see Besfamille et al. (2009) for example.

3 The model

3.1 Basic approach

For a one-period time horizon, a rational[7] and risk-neutral[8] agent whose gross sales revenue is given by y must decide the proportion a of that revenue to conceal from the tax authority, by whom is levied a tax $\tau = (t/100)\, y$ where $0 \leq t \leq 100$.[9]

When evasion is a positive quantity, the agent must face concealment costs, which are denoted by $\varsigma = \varsigma(a, y, c)$ where the parameter c indicates the significance of concealment costs. This function satisfies $\varsigma_a, \varsigma_y, \varsigma_c > 0$ and $\varsigma_{aa} \geq 0$.

The agent perceives a probability $p = p(a, y, \upsilon, \omega, \gamma)$ of being detected. The variables υ, ω, and γ represent, respectively, the influence of behavioral biases, perceived behavior of other agents, and perceived behavior of the tax authority on the aforementioned probability. It is assumed that $p_a, p_y > 0$, $p_{aa} \geq 0$, and $p_\upsilon, p_\omega, p_\gamma < 0$.

If caught, the agent is fined a sum $\chi = \chi(a, y, x)$ for the offense. The parameter x indicates the severity of the penalties legislated. We have $\chi_a, \chi_y, \chi_x > 0$ and $\chi_{aa} \geq 0$.

When evasion is attempted, the subjective expected net revenue can be expressed as:

$$y^e = p\left[(1 - t/100)\, y - \chi - \varsigma\right] + (1 - p)\left[(1 - t/100)\,(y - ay) + ay - \varsigma\right]$$

Or, what is equivalent:

$$y^e = (1 - t/100)\, y + (t/100)\,(1 - p)\, ay - p\chi - \varsigma.$$

Subtracting $(1-t/100)y$ from both sides and denoting $\pi = y_e - (1-t/100)y$ as the expected tax evasion profit, we get:

$$\pi = (t/100)\,(1 - p)\, ay - p\chi - \varsigma. \tag{1}$$

Equation (1) shows that π has three components that distinguish it from $(1-t)y$: Expected additional revenue, expected fine, and concealing costs. We highlight that π=0 if $a = 0$. The agent will choose a^* so as to maximize π. If the agent considers that the decision will neither affect other agents' behavior nor the tax authority's behavior, Kuhn-Tucker optimization with $0 \leq a \leq 1$ and $\pi \geq 0$ as constraints (see the Appendix) yields the following solution:

[7]By this, we that the agent maximizes its subjective expected utility (Savage, 1954). Although decision-making frameworks such as the prospect theory or bounded rationality in any of its forms are available, I believe that the complexity involved in using them outweighs the benefits.

[8]Although it is common knowledge that agents tend to be risk-averse, Arias (2005) has shown that assuming risk neutrality and incorporating concealing costs (which act similarly to a risk premium) leads to similar results as those models that assume risk aversion.

[9]Note that this tax is expressed in terms of the after-tax output value. If a pre-tax charge ρ is modeled, as would be the case with a retail sales tax, replace $t = \rho/(1 + \rho)$.

$$a^* = \begin{cases} 1 & if \quad \pi_a > 0 \ \forall a \in (0,1) \\ \theta & if \quad \pi_a = 0 \ for \ some \ a \in (0,1) \\ 0 & if \quad \pi_a < 0 \ \forall a \in (0,1) \end{cases} \tag{2}$$

where

$$\theta = \frac{1}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{(t/100)\,y}\right].$$

Now, let $z = ay$ represent the amount of tax evasion. Its optimal value is given by:

$$z^* = \begin{cases} 1 & if \quad \pi_a > 0 \ \forall z \in (0,y) \\ \psi & if \quad \pi_a = 0 \ for \ some \ z \in (0,y) \\ 0 & if \quad \pi_a < 0 \ \forall z \in (0,y) \end{cases} \tag{3}$$

where

$$\psi = \frac{y}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{(t/100)\,y}\right].$$

Regarding comparative statics, when y is held constant, the effects of any other variable on a^* or z^* have the same direction. The results for z^* are:

$$\frac{\partial z^*}{\partial x} = -\frac{1}{p_a}\left(\frac{p_a\chi_x + p\chi_{ax}}{t/100}\right) \tag{4a}$$

$$\frac{\partial z^*}{\partial c} = -\frac{1}{p_a}\left(\frac{\varsigma_{ac}}{t/100}\right) \tag{4b}$$

$$\frac{\partial z^*}{\partial t} = \frac{y}{p_a}\left[\frac{\varsigma_a + p_a\chi + p\chi_a}{(t^2/100)\,y}\right] \tag{4c}$$

$$\frac{\partial z^*}{\partial \upsilon} = -\frac{y}{p_a}\left[p_{a\upsilon}\theta + p_\upsilon + \frac{p_{a\upsilon}\chi + p_\upsilon\chi_a}{(t/100)\,y}\right] \tag{4d}$$

$$\frac{\partial z^*}{\partial \omega} = -\frac{y}{p_a}\left[p_{a\omega}\theta + p_\omega + \frac{p_{a\omega}\chi + p_\omega\chi_a}{(t/100)\,y}\right] \tag{4e}$$

$$\frac{\partial z^*}{\partial \gamma} = -\frac{y}{p_a}\left[p_{a\gamma}\theta + p_\gamma + \frac{p_{a\gamma}\chi + p_\gamma\chi_a}{(t/100)\,y}\right]. \tag{4f}$$

When $0 < \theta < 1$, an increase in t, υ, ω or γ, or a decrease in c or x encourages higher tax evasion levels in both absolute and relative terms.

However, when there are variations in y, the effects on a^* and z^* might have different directions, because

$$\frac{\partial a^*}{\partial y} = -\frac{1}{p_a}\left(p_{ay}\theta + p_y + B\right) \tag{5a}$$

$$\frac{\partial z^*}{\partial y} = -\frac{y}{p_a}\left(p_{ay}\theta + p_y + B\right) + \theta, \tag{5b}$$

where B is equal to

$$\frac{\left(\varsigma_{ay} + p_{ay}\chi + p_a\chi_y + p_y\chi_a + p\chi_{ay}\right)y - \left(\varsigma_a + p_a\chi + p\chi_a\right)}{(t/100)\,y^2}.$$

There could be, simultaneously, a decrease in the *proportion* of the gross sales revenue concealed and an increase in the *amount* hidden (or vice versa). Without stronger assumptions, the impact of y on a^* and z^* remains ambiguous.[10]

3.2 Endogenous revenue approach

So far, it has been assumed that the tax rate do not affect gross sales revenue. This may be unrealistic since it ignores either or both possible tax shifting and changes in the relative commodity demand generated by variations in the tax rate. To address this problem, now I consider a scenery where y is affected by t (this is, by tax policy) but not by the decisions of the potential tax evader.

Let $y = y(t, \varepsilon)$ denote the gross sales revenue as a function of the tax rate t and all other non-tax-related factors ε affecting it. The main implication is that now we have $p = p[a, y(t), \upsilon, \omega, \gamma]$, $\chi = \chi[a, y(t)]$, and $\varsigma = \varsigma[a, y(t)]$. Using the chain rule in Equations (2) and (3) to derive θ and ψ with respect to t, the new results are, respectively:

$$\frac{\partial a^*}{\partial t} = -\frac{1}{p_a}\left[(1+\theta)\,p_y y_t + D y_t - \frac{\varsigma_a + p_a\chi + p\chi_a}{(t^2/100)\,y}\left(1 + y_t\frac{t}{y}\right)\right] \tag{6a}$$

$$\frac{\partial z^*}{\partial t} = -\frac{y}{p_a}\left[(1+\theta)\,p_y y_t + D y_t - \frac{\varsigma_a + p_a\chi + p\chi_a}{(t^2/100)\,y}\left(1 + y_t\frac{t}{y}\right)\right] + \theta y_t, \tag{6b}$$

where

$$D = \frac{\varsigma_{ay} + p_{ay}\chi + \chi_y + p_y\chi_a + p\chi_{ay}}{(t/100)\,y}.$$

The effects of a variation in t are unclear, as they largely depend on y_t, for which we cannot make any assumption.[11] However, it is worth noting that a sufficient condition for $a_t^* > 0$ might be $-1 < \eta < 0$, where $\eta = y_t(t/y)$ denotes the elasticity between gross sales revenue and tax rates. Only if $-1 < \eta < 0$ and $\theta/(1-\theta) < p_y y/p_a$ we can ensure z_t^*.

As it is shown in the following section, these ambiguities are far from irrelevant. They shape the relationships between the relative size of the shadow economy, government revenue, and tax rates. Moreover, to a great extent, they determine whether or not a traditional Laffer curve emerges.

4 Aggregate analysis

4.1 Assumptions

Consider a closed economy consisting of n agents, where the total gross sales revenue is denoted by $Y = \sum_{i=1}^{n} y_i$ and a tax $T = (t/100)Y$ is levied on it.

[10]In Section 5.1, I introduce a linear version of the model to study these relationships in further detail.

[11]Even if it is assumed that no variations in relative demand take place when t increases or decreases, y might still be affected in two opposite directions by changes in quantities sold and after-tax prices.

The total amount of evasion is represented by $Z = \sum_{i=1}^{n} z_i$. Evasion is partially discovered by the tax agency, resulting in a sum $(t/100)\sum_{i=1}^{n-j} z_i + \sum_{i=1}^{n-j} C_i$ of additional government revenues, where $n-j$ is the number of agents caught and fined. I assume that it is allocated in its totality to cover the costs of the audits, in order to focus on taxation.

4.2 Government revenue in the presence of tax evasion

4.2.1 Mathematical expressions

Under the assumptions of Section 4.1, government revenue can be expressed as:

$$R = \frac{t}{100}\left(Y - Z\right),$$

If we extract Y from $(Y - Z)$ and use $\phi = Z/Y$ to denote the relative size of the shadow economy, either discovered or not, we can rewrite R as follows:

$$R = \frac{t}{100} Y \left(1 - \phi\right). \tag{7}$$

Regarding the properties of ϕ as a function, its first and second derivative with respect to t are, respectively:

$$\frac{\partial \phi}{\partial t} = \frac{1}{Y}\left(Z_t - \phi Y_t\right) \tag{8a}$$

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{Y}\left[-\frac{Y_t}{Y}\left(Z_t - \phi Y_t\right) + \left(Z_{tt} - \phi Y_{tt}\right) - \phi_t Y_t\right]. \tag{8b}$$

If t is taken as given, the sufficient condition for $\phi_t > 0$ is $Z_t > \phi Y_t$, since $0 \leq \phi \leq 1$. According to Equations (6a) and (6b), it is guaranteed if $0 < \eta_i < 1$ and $\theta_i/(1-\theta_i) < p_{y(i)} y_i / p_{a(i)}$ $\forall i = 1, 2, ..., n$. Regarding $\phi_{tt} > 0$, ensuring it requires $Z_t > \phi Y_t$, $Z_{tt} > Y_{tt}$, and $Y_t \leq 0$.

4.2.2 Breaking down the effects of a tax cut

Relying on how government revenue has been defined, the effects of a tax cut in the presence of tax evasion can be broken down in an alternative[12] way.

According to Laffer (2004), if there is no tax evasion, reducing the tax rate will have two opposing effects on government revenue. It will reduce them per unit of the tax base (arithmetic effect), but, at the same time, it will increase the tax base (economic effect).

When tax evasion is present and the tax rate falls from t_0 to t_1, although Y increases in δ units, which is the *gross economic effect*, a portion $\phi_1 \delta$ of this increase is not reflected in a higher effective tax base since the agents conceal it from the authorities (see Equation (7)), hence the *net economic effect* is given by $(1 - \phi_1)\delta$.

[12] The distinction between the compliance effect and the income effect has probably been the most accepted in the tax evasion literature. See (Takáts & Papp, 2008).

4.3 First-order condition, second-order condition and uniqueness of the maximum

The first and second-order conditions that a tax rate t^* must satisfy in order to maximize $R = (t/100)(Y - Z)$ are, respectively:

$$\frac{\partial R}{\partial t} = \frac{1}{100}\left[(Y - Z) + t\left(Y_t - Z_t\right)\right] = 0$$
$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{100}\left[2\left(Y_t - Z_t\right) + t\left(Y_{tt} - Z_{tt}\right)\right] < 0.$$

Note that $Y \geq Z$ $\forall t \in [0, 100]$. Now, if $Z_t > Y_t$ and $Z_{tt} > Y_{tt}$ are satisfied $\forall t \in [0, 100]$, then both expressions $Y - Z$ and $-t(Y_t - Z_t)$ will always be non-negative, but the former will decrease and the latter will increase as t grows. Under those circumstances, $-t(Y_t - Z_t)$ will be zero when $t = 0$ and positive when $t = 100$. Provided $Y - Z$ positive when $t = 0$ and null when $t = 100$, there can only exist one critical point $t^* \in (0, 100)$ where the first-order condition is met, which will also be a maximum because the second-order condition will also be true. Therefore, a traditional Laffer curve for government revenue from gross sales revenue will exist.[13] Furthermore, it is possible to rewrite the first-order condition more elegantly by solving for t and considering $\phi = Z/Y$:

$$t^* = \frac{Y\left(1 - \phi\right)}{Z_t - Y_t}.$$

Replacing with this solution in Equation (7) yields R^*, which is the maximum potential government revenue in nominal terms:

$$R^* = \frac{Y^2\left(1 - \phi\right)^2}{100\left(Z_t - Y_t\right)}$$

4.4 The nominality issue

In could be argued that t^* may not maximize government revenue in real terms due to the impact of tax rate adjustments on prices. Since the model does not deal with prices, we can only aim to maximize government revenue as a proportion of the total gross sales revenue of the economy (henceforth in proportional terms). Consider $r = R/Y = (t/100)(1-\phi)$. In order to maximize this function, following first and second-order conditions must be satisfied:

$$\frac{\partial r}{\partial t} = \frac{1}{100}\left[(1 - \phi) - t\phi_t\right] = 0$$
$$\frac{\partial^2 r}{\partial t^2} = -\frac{1}{100}\left(2\phi_t + t\phi_{tt}\right) < 0.$$

[13] It must be highlighted, however, that without further assumptions about the characteristics of the individual functions involved or the distribution of the agents, it is not possible to ensure that those conditions will be held.

Provided $Y > Z$ if $t = 0$ and $Y = Z$ if $t = 100$, then $1 - \phi > 0$ when $t = 0$ and $1 - \phi = 0$ when $t = 100$. If in addition $\phi_t > 0$ and $2\phi_t > (-t\phi_{tt})$ $\forall t \in [0, 100]$, which requires $Z_{tt} > Y_{tt}$ and $Y_t \leq 0$ whenever $0 \leq t \leq 100$ (see Equations (8a) and (8b)), r will have a unique maximum $t^{**} \in (0, 100)$.[14] This maximum may be expressed as:

$$t^{**} = \frac{(1 - \phi)}{\phi_t}.$$

In this scenario, since $r = (t/100)(1 - \phi)$, the maximum potential government revenue in proportional terms is given by:

$$r^{**} = \frac{(1 - \phi)^2}{100\phi_t}.$$

4.5 Implications

Various results from the aggregate analysis, which are valid in both nominal and proportional terms, ought to be highlighted. The proofs for each of them can be found in the Appendix.

Proposition 1 *In the presence of tax evasion, a shadow economy whose relative size increases whenever the tax rate is raised is a necessary condition in order to guarantee the existence of a traditional Laffer curve, yet not a sufficient one.*

Proposition 2 *In the presence of tax evasion, for any given tax rate, government revenue is always lower than in a no-evasion situation. Additionally, it tends to decrease as tax evasion increases due to non-revenue-related factors. Therefore, the position of the curves $R(t)$ and $r(t)$ can be affected by the evasion control policy.*

Proposition 3 *In the presence of tax evasion, if the existence of a traditional Laffer curve is guaranteed, then both the tax rate for which it is maximum and the maximum potential government revenue are lower compared to the no-evasion situation.*

To the best of my knowledge, although some models of indirect tax evasion (Kanniainen & Pääkkönen, 2004; Kotamäki, 2017; Palda, 1998; Vasilev, 2018; Vogel, 2012) and linear income tax evasion (both old (Feige & McGee, 1983; Peacock & Shaw, 1982; Ricketts, 1984) and not-so-old (Besfamille, 2008; Busato & Chiarini, 2013; Chang et al., 1999; Méder et al., 2012)) have partially provided those results either explicitly or implicitly, a systematic and simultaneous presentation of them have not taken place until now.

[14] If $Z_t > Y_t$, $Y_t < 0$, and $Z_{tt} > Y_{tt}$ are satisfied $\forall t \in [0, 100]$, then both expressions $1 - \phi$ and $t\phi_t$ will always be non-negative, but the former will decrease and the latter will increase as t grows. Under those circumstances, $t\phi_t$ will be zero when $t = 0$ and positive when $t = 100$. Provided $1 - \phi$ positive when $t = 0$ and null when $t = 100$, there can only exist one critical point $t^{**} \in (0, 100)$ where the first-order condition is met, which will also be a maximum because the second-order condition will also be true.

5 Simulation

5.1 A particular version of the model

To avoid unnecessary complexity, a linear version of the model (with a closed solution) is used in the simulation. Although its stronger assumptions, it allows for an interesting interaction between y and γ. For each agent, let $p_i = z_i/(\upsilon_i \omega_i y_i^{\gamma} + z_i)$ be the perceived probability of detection, $\chi_i = x z_i$ be the fine that would be imposed for the offense if detected, and $\varsigma_i = c_i z_i$ be the concealment costs, where $x, \upsilon_i, \omega_i, c_i > 0$, and $\gamma < 1$. Note that p_i is naturally bounded between 0 and 1, and the assumption $p_{aa(i)} \geq 0$ for all $i = 1, 2, ..., n$ has been relaxed.

Why $\gamma < 1$? In this simplified version of the model, the partial derivative of p_i with respect to y_i is:

$$\frac{(1-\gamma)\, a_i \upsilon_i \omega_i y_i^{\gamma}}{\left(\upsilon_i \omega_i y_i^{\gamma} + a_i y_i\right)^2}.$$

As the assumption $p_{y(i)} < 0 \, \forall i = 1, 2, ..., n$ ought not to be forgone, the condition $\gamma < 1$ must be satisfied. Keeping it seems reasonable because it implies that the perceived likelihood of being caught for a fixed *proportion* of evasion increases as gross sales revenue raises.[15] It is worth noting that if $\gamma < 0$, the perceived likelihood of being caught for a fixed *amount* of evasion increases as gross sales revenue raises.[16]

Replacing in Equation (1) with the stronger assumptions yields $\pi_i = (t/100 - c_i) a_i y_i - [(t/100 + x) a_i^2 y_i^2]/(\upsilon_i \omega_i y_i^{\gamma} + a_i y_i)$.[17] Doing the same replacement in Equation (3) we obtain:

$$z_i^* = \begin{cases} y_i & if \quad t/100 \geq h_i \\ \psi_i & if \quad c_i < t/100 < h_i \\ 0 & if \quad t/100 \leq c_i \end{cases}$$

where

$$\psi_i = \left(\sqrt{\frac{x + t/100}{x + c_i}} - 1\right) \upsilon_i \omega_i y_i^{\gamma}$$

and

$$h_i = (x + c_i) \left(\frac{y_i^{1-\gamma}}{\upsilon_i \omega_i}\right)^2 + 2\,(x + c_i) \left(\frac{y_i^{1-\gamma}}{\upsilon_i \omega_i}\right) + c_i$$

[15] Roughly speaking, this means that when a small business and a *blue chip* company conceal the same proportion of their revenues, all other variables equal, the former will always perceive a lower probability of detection than the latter.

[16] Denoting those elements as $\tilde{z}_i$ and $\tilde{p}_i$ to avoid any possible misunderstanding, we have $\partial \tilde{p}_i / \partial y_i = -\gamma \left(\upsilon_i \omega \tilde{z}_i y_i^{\gamma}\right) / y_i \left(\upsilon_i \omega_i y_i^{\gamma} + \tilde{z}_i\right)^2$. In intuitive terms, $\gamma < 0$ means that a small business will always perceive a lower probability of detection than a *blue chip* company were they to conceal the same quantities from the tax authority, all other variables equal.

[17] Under these assumptions and taking h_i as defined at the end of this page, $\pi_{a(i)} > 0 \; \forall a \in (0, 1)$ only if $t/100 > h_i$, $\pi_{a(i)} < 0 \; \forall a \in (0, 1)$ only if $t/100 < c_i$, and $\exists! a \in (0, 1)$ such that $\pi_{a(i)} = 0$ only if $c_i < t/100 < h_i$. Note that if $t/100 = c_i$, then $\pi_{a(i)} = 0$ when $a = 0$, and if $t/100 = h_i$, then $\pi_{a(i)} = 0$ when $a = 1$. Additionally, $\pi_{aa(i)} < 0 \; \forall a \in [0, 1]$.

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If $0 < \gamma < 1$, the optimal amount of evasion decreases in relative terms but increases in absolute terms as gross sales revenue does. If $\gamma < 0$, the optimal amount of evasion decreases in both relative and absolute terms as gross sales revenue increases.

5.2 Distributional assumptions

For every agent $i = 1, 2, ..., n$, the relationship between gross sales revenue and the tax rate is assumed to be given by $y_i = q_i t^2 + s_i t + \tilde{y}_i$, where $q_i, s_i, \tilde{y}_i$ are constants, and $\tilde{y}_i > 0$.

We assume that the variables $\tilde{y}_i$, υ_i, ω_i, and c_i follow a multivariate normal distribution $\mathbf{W} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\mathbf{W} = \begin{pmatrix} \tilde{y}_i \\ \upsilon_i \\ \omega_i \\ c_i \end{pmatrix}, \qquad \boldsymbol{\mu} = \begin{pmatrix} 5000 \\ 6000 \\ 1800 \\ 0.01 \end{pmatrix}, \qquad \boldsymbol{\Sigma} = VarCov(\mathbf{W}).$$

For a meaningful characterization of $\boldsymbol{\Sigma}$, define $\boldsymbol{\sigma}(\mathbf{W})$ as the vector of standard deviations of $\mathbf{W}$ and $Corr(\mathbf{W})$ as the correlation matrix of $\mathbf{W}$. Therefore, we have:

$$\boldsymbol{\sigma}(\mathbf{W}) = \begin{pmatrix} 1000 \\ 500 \\ 300 \\ 0.001 \end{pmatrix}, \qquad Corr(\mathbf{W}) = \begin{pmatrix} 1 & -0.6 & -0.4 & 0.7 \\ -0.6 & 1 & 0.8 & -0.2 \\ -0.4 & 0.8 & 1 & -0.3 \\ 0.7 & -0.2 & -0.3 & 1 \end{pmatrix}.$$

Two different cases are presented. In the first one (henceforth Case 1) it is assumed that $q_i = -m_i$, while in the second one (henceforth Case 2) the assumption is $q_i = m_i$. In both cases, $m_i \sim logN(1, 0.25)$ and $s_i \sim logN(1, 0.25)$.

5.3 Results

$n = 1000$ has been set for data generation. Descriptive statistics of the behavioral parameters obtained are presented in Table 1.

Figure 1 shows the relationship between total gross sales revenue and the rate for each case. In Case 1, Y decreases whenever t increases, while the opposite occurs in Case 2. Although these curves determine, to a great extent, how Z, ϕ, R, and r are related to t, note that these relationships cannot be affected by changes in the parameters of the evasion control policy (γ and x).

In Figure 2, the results for Case 1 are shown. The level of tax evasion is inversely related to the variable t, following an inverse U-shaped pattern. The share of the shadow economy tends to increase as the tax rate raises, and traditional Laffer curves in both nominal and proportional terms are observed despite the necessary conditions for their existence not being met. For any given tax rate, a reduction in γ or an increase in x can simultaneously reduce total

Variable	Mean	SD	Min	Max
tilde_y_i	5,009.044	978.244	2,291.587	8,216.046
upsilon_i	5,978.199	520.115	4,334.780	7,715.148
omega_i	1,790.747	310.661	717.115	2,795.972
c_i	0.010	0.001	0.007	0.013
m_i	1.024	0.262	0.456	2.367
s_i	1.040	0.261	0.442	2.532

Table 1

tax evasion, decrease the share of the shadow economy, and increase government revenue in both nominal and proportional terms.

Figure 3 allows us to see the results for Case 2. As the tax rate increases, the level of tax evasion also increases. The relative size of the shadow economy shows an inverse U-shaped relationship with the tax rate, and no Laffer curve arises in either nominal or proportional terms. It is to be highlighted that for any given tax rate, it is again possible to simultaneously reduce total tax evasion, decrease the relative size of the shadow economy, and increase government revenue (in both nominal and proportional terms) by setting higher values for x or lower values (higher in absolute value) for γ.

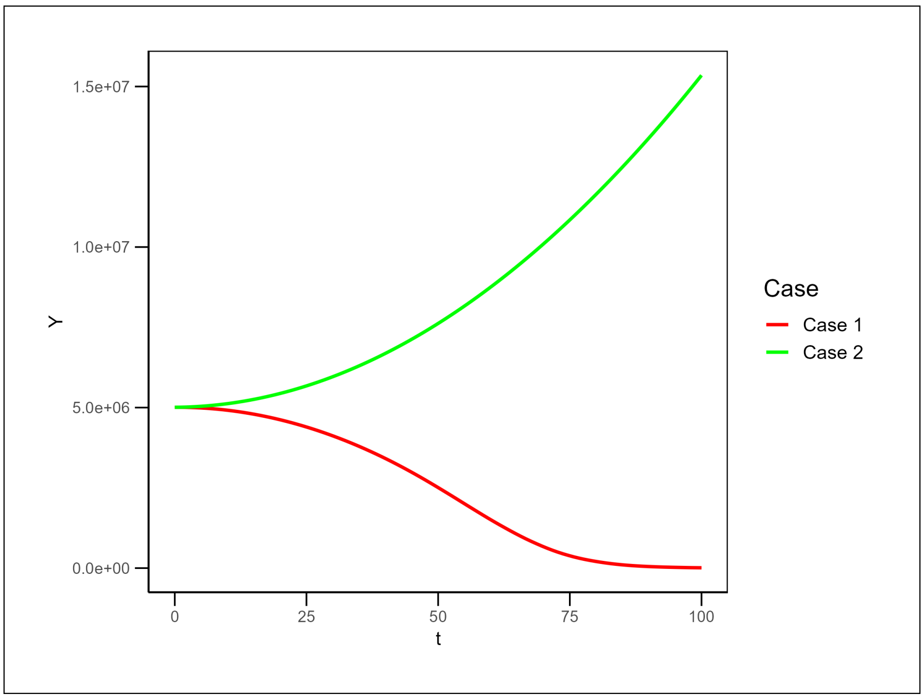
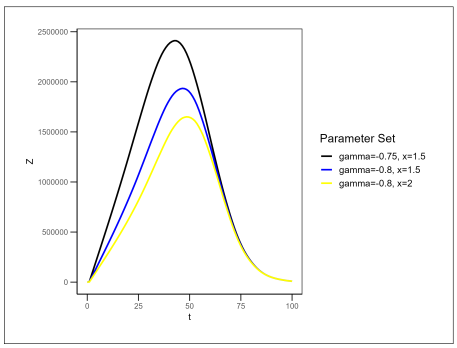
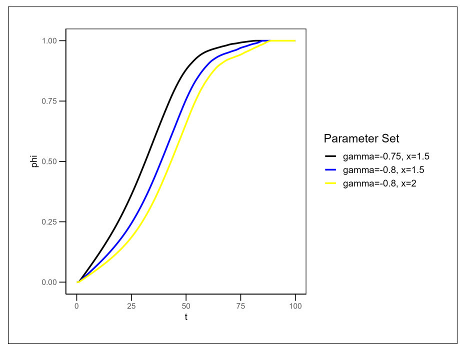


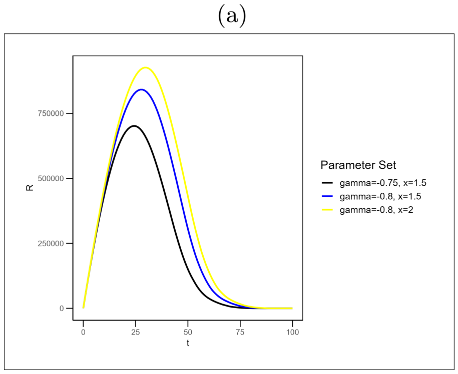
Figure 1



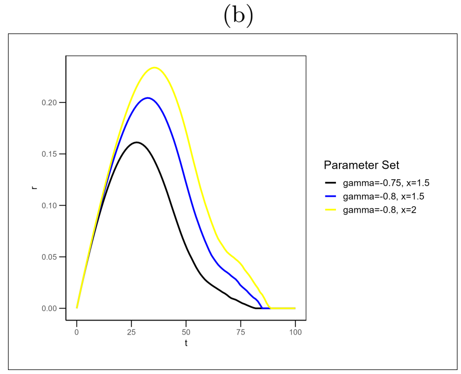
(a)



(b)

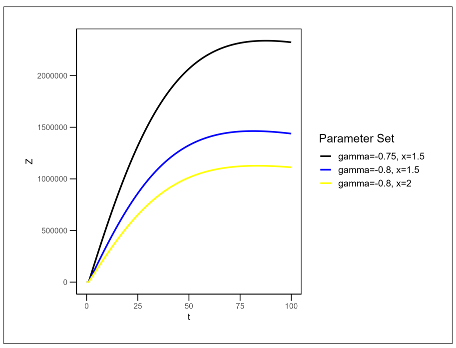


(c)

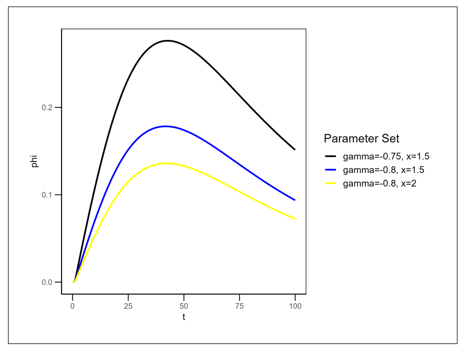


(d)

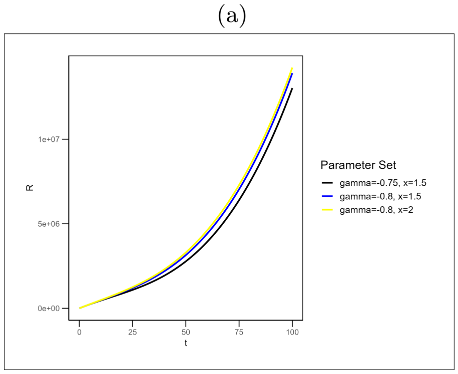
Figure 2



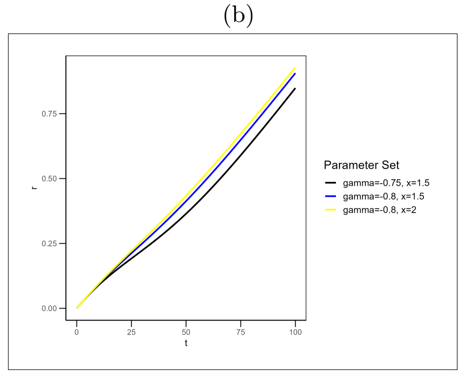
(a)



(b)



(c)



(d)

Figure 3

6 Concluding remarks

A model that recognizes the uncertainty faced by the decision-maker when trying to evade a sales tax has been developed. Its most important feature is that beliefs about the probability of being detected and caught are influenced by behavioral biases, government activity, and other agents' behavior. This framework has enabled an analysis of the effects of tax rates on the shadow economy and government revenue. Special attention has been given to the conditions under which, in the presence of indirect tax evasion, a traditional Laffer curve (in either nominal or proportional terms) emerges. Moreover, the implications of its existence have been studied in detail.

It is important to recognize that certain issues have remained beyond the scope of the presented mathematical formalization. The potential impact of tax evasion on gross sales revenue, whether through adjustments in prices or quantities, has not been considered. The framework does not go beyond one single period of time, and it neglects the influence of moral values or the possible substitution between tax avoidance and tax evasion.[18] Lastly, oversimplifying assumptions might have been made about tax audits, such as binary outcomes for each level of evasion or perfect coverage of their cost by the additional income that they entail.[19]

Notwithstanding the limitations of this theory, its validity is subject to future empirical testing. However, it will be challenging in a field where the lack of data and its reliability have always been a problem (Alm, 2019; Andreoni et al., 1998; Cowell, 1985). One potential approach for future empirical research is to collect data through experimental economics.

[18] In recent work (degl'Innocenti & Rablen, 2017; degl'Innocenti et al., 2022), simultaneous decisions of tax evasion and tax avoidance have been modeled. This could be an interesting extension.

[19] Sanyal et al. (2000) present a simple framework in which corruption leads to non-binary outcomes, while in Arozamena et al. (2008), the audit cost determines whether or not the government commits to enforcing the tax law. Future extensions in any of these directions seem both plausible and worthwhile.

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Annex

Individual variables

y: Gross sales revenue.
τ: Tax levied on gross sales revenue.
t: Tax rate as a proportion of gross sales revenue (in after-tax terms).
a: Proportion of gross sales revenue concealed.
a^*: Optimal proportion of gross sales revenue concealed (the interior solution is denoted as θ).
z: Amount of gross sales revenue concealed.
z^*: Optimal amount of gross sales revenue concealed (the interior solution is denoted as ψ).
ς: Concealment costs.
c: Indicates the significance of concealment costs.
p: Perceived probability of detection.
υ: Influence of behavioral biases on the perceived probability of detection.
ω: Influence of other agents' perceived behavior on the perceived probability of detection.
γ: Influence of the perceived behavior of the tax authority on the perceived probability of detection.
χ: Fine imposed if evasion is detected.
x: Indicates the severity of the penalties legislated.
y^e: Expected net revenue.
π: Expected tax evasion profit.
η: Elasticity of gross sales revenue with respect to the tax rate.
ε: Non-tax related factors affecting gross sales revenue.

Aggregate variables

Y: Total gross sales revenue.
T: Total tax levied on gross sales revenue.
Z: Total amount of tax evasion.
ϕ: Relative size of the shadow economy.
R: Government revenue.
R^*: Maximum potential government revenue in nominal terms.
t^*: Tax rate that maximizes government revenue in nominal terms.
r: Government revenue as a proportion of total gross sales revenue.
r^{**}: Maximum potential government revenue as a proportion of total gross sales revenue.
t^{**}: Tax rate that maximizes government revenue as a proportion of total gross sales revenue.

Notation

Unless a remarkable result is to be displayed, the partial derivative of one variable with respect to another is denoted by using the latter as a subscript. For example, p_y represents the first derivative of the perceived probability of detection with respect to gross sales revenue.

From the aggregate analysis onward, I use the sub-index i to indicate that I am referring to an individual variable. For instance, y_i denotes the gross sales revenue of agent i. To avoid misunderstandings, when referring to the partial derivative of an individual variable with respect to another individual variable, I use a subscript containing the latter accompanied by i in parentheses. As an example, $p_{y(i)}$ is the first derivative of the perceived probability of detection with respect to gross sales revenue for agent i.

Appendix

Kuhn-Tucker Optimization Problem

The problem is:

$$\max_a \quad \pi = \frac{t}{100}(1-p)ay - p\chi - \varsigma \quad \textbf{s. t.} \quad 0 \leq a \leq 1, \quad \pi \geq 0.$$

The Lagrangian function that we need to maximize is written in the following way:

$$\mathcal{L}(a, \lambda_1, \lambda_2, \lambda_3) = (1+\lambda_3)\pi + \lambda_1 a + \lambda_2(1-a),$$

where λ_1, λ_2, and λ_3 are the Lagrange multipliers associated with each of the constraints.

The conditions of stationarity, primal feasibility, dual feasibility, and complementary slackness are, respectively:

$$\frac{\partial \mathcal{L}}{\partial a} = (1+\lambda_3)\pi_a + \lambda_1 - \lambda_2 = 0 \tag{A1}$$

$$0 \leq a \leq 1, \quad \pi \geq 0 \tag{A2}$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq 0 \tag{A3}$$

$$\lambda_1 a = 0, \quad \lambda_2(1-a) = 0, \quad \lambda_3 \pi = 0. \tag{A4}$$

This problem has three possible solutions:

i) Interior solution: It occurs when $\pi_a = 0$ for some $a \in (0,1)$. Considering (A3) and (A4), we require $\lambda_1, \lambda_2 = 0, \lambda_3 \geq 0$. Note that (A1) will be satisfied $\forall \lambda_3 \in \mathbb{R}_+$. As demonstrated later on, it consists on a unique maximum where $\pi > 0$ is satisfied.

ii) Total evasion solution: It arises if $\pi_a > 0 \ \forall a \in (0,1)$. Given the constraints from (A2), the maximum feasible value of π is reached when $a = 1$, where $\pi > 0$ is ensured since $\pi = 0$ when $a = 0$. Due to (A3) and (A4), we need $\lambda_1, \lambda_3 = 0, \lambda_2 \geq 0$. Regarding (A1), as $\pi_a \geq 0$ for $a = 1$, $\exists \lambda_2 \in \mathbb{R}_+$ such that $\pi_a - \lambda_2 = 0$.

iii) No-evasion solution: In this case, $\pi_a < 0 \ \forall a \in (0,1)$. Considering the constraints from (A2), the maximum feasible value of π is reached when $a = 0$, resulting in $\pi = 0$. Because of (A3) and (A4), it is necessary that $\lambda_2 = 0$ and $\lambda_1, \lambda_3 \geq 0$. In reference to (A1), given that $\pi_a \leq 0$ for $a = 0$, $\exists \lambda_1, \lambda_3 \in \mathbb{R}_+$ such that $(1+\lambda_3)\pi_a + \lambda_1 = 0$.

Now, I prove that if an interior solution θ exists, it is a unique maximum that satisfies $\pi > 0$. Taking into account that

$$\frac{\partial \pi}{\partial a} = \frac{ty}{100}(1 - p - p_a a) - p\chi_a - p_a\chi - \varsigma_a.$$

We have

$$\frac{\partial \pi}{\partial a} = 0 \iff a = \theta = \frac{1}{p_a}\left[(1-p) - \frac{\varsigma_a + p_a\chi + p\chi_a}{(t/100)y}\right].$$

Also considering that

$$\frac{\partial^2 \pi}{\partial a^2} = -\frac{1}{p_a}\left(p_{aa}\theta + \frac{C_{aa} + p_{aa}X + 2p_aX_a + pX_{aa}}{(t/100)\,y}\right) - 1 < 0 \quad \forall \theta \in (0,1),$$

it is clear that θ is a maximum. It is also unique because if two critical points θ_1, θ_2 were to exist, at least one of them should be either a minimum or an inflection point, which would imply $\pi_{aa} \geq 0$ for some $\theta \in (0,1)$ (i.e., a contradiction). Recalling that $\pi = 0$ when $a = 0$, it is easy to conclude that $\pi > 0$ if $a = \theta$.

Finally, the solution to the Kuhn-Tucker optimization problem can be expressed in the following way:

$$a^* = \begin{cases} 1 \quad if & \pi_a > 0 \ \forall a \in (0,1) \\ \theta \quad if & \pi_a = 0 \ for\ some\ a \in (0,1) \\ 0 \quad if & \pi_a < 0 \ \forall a \in (0,1) \end{cases}$$

where

$$\theta = \frac{1}{p_a}\left[(1-p) - \frac{C_a + p_aX + pX_a}{(t/100)\,y}\right].$$

Proof of Propositions 1-3

Proof of Proposition 1

In nominal terms: *Assume $Y - Z > 0$ when $t = 0$, and $Y = Z = 0$ when $t = 100$. The condition $Z_t > Y_t$ $\forall t \in [0,100]$ necessarily implies $Z_t > \phi Y_t$ $\forall t \in [0,100]$ since $0 \leq \phi \leq 1$, which means $\phi_t > 0$ for all t such that $0 \leq t \leq 100$. However, this cannot ensure $Z_{tt} > Y_{tt}$ $\forall t \in [0,100]$, thus R may have more than one critical point in $0 < t < 100$.*

In proportional terms: *The demonstration is essentially the same. $\phi_t > 0$ $\forall t \in [0,100]$ can only satisfy the first-order condition, not the second-order one.*

Proof of Proposition 2

In nominal terms: *Considering $R = t(Y - Z)/100$, it is clear that $tY/100 > t(Y-Z)/100$ as long as $0 < Z \leq Y$. We also have $R_Z = -t < 0$. Without loss of generality, consider the case for γ. As we have stated, $z^*_{\gamma(i)} > 0$ $\forall i = 1, 2, .., n$, hence $Z_\gamma = \sum_{i=1}^{n} z^*_{\gamma(i)} > 0$. Using the chain rule, $R_\gamma = -tZ_\gamma < 0$.*

In proportional terms: *The demonstration is analogous since $r = t(1 - Z/Y)$.*

Proof of Proposition 3

In nominal terms: *The no-evasion situation is a particular case in which ensuring the existence of a traditional Laffer curve requires $Y_t, Y_{tt} < 0$ $\forall t \in [0,100]$ since $Z, Z_t, Z_{tt} = 0$ $\forall t \in [0,100]$. For the comparison of this case to another*

one where $Z > 0$ $\forall t \in [0, 100]$ we need $Z_t, Z_{tt} > 0$ $\forall t \in [0, 100]$, otherwise a traditional Laffer curve would not exist. Now I prove the proposition for:
i) t^*: Let $(t^*)_0 = -Y_0/(Y_t)_0$ if $Z = 0$, and $(t^*)_1 = Y_1(1-\phi)/[(Z_t)_1 - (Y_t)_1]$ if $Z > 0$. Assume $(t^*)_1 > (t^*)_0$. Since $Y_t < 0$ and $0 < \phi \leq 1$ $\forall t \in [0, 100]$, we have $Y_0 > Y_1(1-\phi)$. Therefore, $-(Y_t)_0 > (Z_t)_1 - (Y_t)_1$ necessarily needs to be met, which is equivalent to $-(Y_t)_0 + (Y_t)_1 > (Z_t)_1$. Because $Y_{tt} < 0$ $\forall t \in [0, 100]$, $-(Y_t)_0 + (Y_t)_1 < 0$ holds. In consequence, $(Z_t)_1 < 0$ is required for $(t^*)_1 > (t^*)_2$, which is a contradiction.
ii) R^*: Define $(R^*)_0 = -(Y_0)^2/100(Y_t)_0$ for the no-evasion situation and $(R^*)_1 = (Y_1)^2(1-\phi)^2/100[(Z_t)_1 - (Y_t)_1]$ for the other case. The same previous contradiction will be encountered if it is assumed that $(R^*)_2 > (R^*)_1$.
In proportional terms: *A traditional Laffer curve in proportional terms cannot exist in the no-evasion situation because $\phi, \phi_t, \phi_{tt} = 0$ $\forall t \in [0, 100]$, thus there are corner solutions for both t^{**} and r^{**}. Having said that, I prove the proposition for:*
i) t^{**}: Let $(t^{**})_0 \approx 100$ if $Z, Z_t, Z_{tt} = 0$ $\forall t \in [0, 100]$, and $(t^{**})_1 = (1-\phi)/\phi_t$ for any other case where a traditional Laffer curve in proportional terms exists. Solving the algebra, $(t^{**})_0 > (t^{**})_1$ only if $(1-\phi)/\phi_t < 100$, which is necessarily true when a traditional Laffer curve in proportional terms exists since it implies $(t^{**})_1 = [(1-\phi)/\phi_t] \in (0, 100)$.
ii) r^{**}: Let $(r^{**})_0 \approx 1$ denote the no-evasion situation and $(r^{**})_1 = (1-\phi)^2/100\phi_t$ any other case where a traditional Laffer curve in proportional terms exists. $(r^{**})_0 > (r^{**})_1$ holds as long as $100/(1-\phi) > (1-\phi)/\phi_t$, which is always satisfied when $0 < (1-\phi)/\phi_t < 100$ because $0 < \phi \leq 1$.