Corrupt Voting: Information and Electoral Accountability*

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Abstract

Does the ability of the electorate to replace corrupt politicians deter corruption? This paper analyzes the limitations of electoral accountability. We show that if the electorate cannot commit elections offer no defense against corruption. However, when a commitment technology exists, the electorate can strategically choose to remove only those caught taking bribes. This incentivizes corrupt politicians to pass up bribe opportunities for which the value is small. We then examine how improved monitoring can impact outcomes and show that increasing information quality does not always benefit the electorate.

1 Introduction

Does the ability of the electorate to replace corrupt politicians deter corruption? We know from a variety of theoretical and empirical work that in a political competition between groups larger groups have an advantage in elections and smaller groups an advantage in lobbying (Leech 2010). Naturally small groups then seek to undermine electoral results by lobbying and bribing politicians once in office. In the worst case

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democracy is meaningless and small lobbying groups run the show, as Gilens and Page (2014) suggests might be the case in the U.S. There are, however, two things, that give some hope: first, the ability to remove corrupt politicians can improve incentives for politicians to act in the voter's interest. Second, voters have an incentive to elect idealists who are hard to bribe.

We explore these issues in a simple dynamic model of a majority party - the electorate - modeled as a single rational voter who chooses a politician. Politicians can be of two types: honest politicians who always act in the electorate's best interest (idealists) and corrupt politicians. We allow for the possibility that the politicians' type changes from one period to the next. In particular idealism is not ex ante observable, and deteriorates over time so that politicians who came to office with the best of intentions may become corrupt. An example might be Barney Frank who served in the U.S. House of Representatives for 32 years. Originally considered an idealistic and incorruptible reformer (and opponent of big banks) he ultimately lost office after it was discovered he was implicitly taking bribes from the banking industry (see the Wikipedia article for details).

In our formal model there is a repeated game. We make the key assumption that following an election the electorate learns whether or not the politician in office is corrupt. Hence, in the next election, politicians can be fired based on whether or not they are corrupt. Our question is whether, prior to a new election, can corrupt politicians be induced not to take bribes? Specifically, during their term in office a politician has a corruption opportunity: this is an action against the electorate's interest but results in the politician receiving a bribe. It is characterized by the size of the bribe and the chances of being caught. After the bribe is (or is not) observed an election takes place. The key point is that when the election takes place the electorate knows whether or not the politician is corrupt but has only observed the bribe with some probability. Never-the-less, since the bribe is sometimes observed, they may impose discipline on a (known) corrupt politician by replacing them if a bribe is observed but not otherwise.

Following the election if the politician is replaced a new office holder is randomly drawn. If the politician remains in office and is honest there is a chance they become corrupt. We assume, however, that this probability is small enough that it is better to keep an honest politician than to drawn anew.

In the model two mechanisms are at work: reputation for honesty and reelection. The electorate thus rationally balances out the chances of a replacement politician being honest with saving itself the cost of replacement while risking a corruption event. But they also understand the effect of their strategy on future politicians' behavior. We ask: Under what conditions may we expect corrupt politicians to behave honestly by passing up bribing opportunities? The answer is twofold. Without commitment never. This is because the electorate's current action does not influence the politician's behavior, hence the electorate does not have any incentive to deviate from her static best response. In contrast, if the electorate can commit to pure strategies then the corrupt politician could prefer to pass up small bribes in order to be in office when a large one arrives. Depending on parameter values, we find that in equilibrium the corrupt politicians either get fired always, or whenever they are caught taking a bribe.

Our analysis emphasizes the importance of a cost to the electorate of getting rid of the incumbent. Changing governments usually entails transition costs (learning, adaptation, etc.) Relatedly, Gersbach, Muller, and Tejada (2019) emphasize the costs of the policy changes that usually come with a changing government. The existence of such costs will be a factor favoring stability and thus will diminish the attrativeness of electoral accountability.

In the commitment equilibrium the global level of corruption may also influence the electorate's behavior. The more likely that a newly elected politician is corrupt, the less likely that observed actions of corruption will go unpunished. In the commitment case we also study the effect of improving information technology. We find that the electorate may prefer a worse information technology. This is because improvements in public auditing can force too many replacements and the electorate may avoid the implied costs by randomly forgiving some acts of corruption.

Literature Review

The literature on dynamic political agency that envisions the electorate as delegating authority on political officers probably starts with Ferejohn (1986). Models in this class mostly consider the choice by the politician of an unobservable policy in the real line with a stochastic, observable, outcome for the electorate also on the real line; politicians incur a private cost in exerting effort, which increases the possibility of better outcomes for the electorate. Ferejohn (1986) assumes that the electorate can commit every period to a retrospective performance-based rule, and studies an equilibrium where the electorate keeps the incumbent if and only if performance is above some threshold. Duggan and Martinelli (2017) point out that should the electorate move after the politician, her past performance would not matter in its decision. The electorate would consider challengers and the incumbent as similar and thus would be indifferent among them all. This would leave it indifferent also among all retention policies. Committing to any particular one would be equivalent to selecting equilibria. Duggan and Martinelli (2017) move on to characterize the electorate-best equilibrium, in which the incumbent exerts high effort. Crucially, neither paper considers the possibility of hiring an idealist (which would dissolve the mentioned universal indifference result), the existence of a cost of getting rid of politicians in office, nor the stochastic nature of corruption opportunities in office, all of which play a key role in our argument.

On the other hand, Banks and Sundaram (1998) study an equilibrium in which the electorate uses an incentive strategy without commitment. They achieve this by mixing moral hazard with adverse selection. As the incumbent type is unknown in their setting, her performance is informative of her type. Hence, the provision of incentives to the politician is a byproduct of the electorate's aim of selecting better politicians. This is unlikely to produce the electorate-best incentives though. Indeed, Besley and Smart (2007) delve into the trade off between incentives and selection that the electorate faces when it is able to choose information quality.

An important result of our work, as we have observed above, is that in our model, the electorate may prefer a less advanced information technology. This contradicts intuition and seems to go against the spirit of the "informativeness principle" proposed by Holmström (1979) and Shavell (1979), which suggests that more information is not detrimental to the principal. However, there are cases where this principle does not hold. One possible reason why more information could be detrimental to the electorate is when more information for the principal is also accessible to a third party that could use it against the electorate (Prat 2005). In our model there are no third-party agents so this effect is not present.

Another possible reason is that revealing more information about the agent might cause the agent's interests to become less aligned with those of the principal. Dewatripont, Jewitt, and Tirole (1999) give examples where the agent works harder when the principal receives signals about the agent's performance rather than observing her actual performance. Holmström (1999) points out that better information about the agent's type reduces the agent's incentives to work hard in order to prove their worth. Prat (2005), in a career concern model, analyzes the case where revealing more information about the agent causes the agent to ignore valuable private information. Crémer (1995), in a model with renegotiation, suggests that the principal's utility may decrease with the quality of the signal because it makes the commitment to non-renegotiation less credible. In our model, the agent's type is known while revealing more information about her action aligns her incentives better with the interests of the electorate.

On the empirical side, there are also papers that study the relationship between information and accountability. Ferraz and Finan (2008) demonstrate that audit reports significantly impact incumbents' electoral performance. In a later study, Ferraz and Finan (2011) find that mayors with reelection incentives tend to be less corrupt, and that the likelihood of corruption detection strengthens the effect of accountability. On the other hand, Boas, Hidalgo, and Melo (2019) explain that in real life, accountability does not have the same impact as it does in controlled experiments. They highlight that corruption accusations often lack specifics, that the campaign context limits the effectiveness of information, and that local factors such as attitudes toward political dynasties influence voter behavior. In their study of Brazil, they observed that voters may punish corrupt mayors in hypothetical scenarios, but not in real elections, due to concerns over local issues like employment and healthcare services. Hence, the evidence of electoral accountability being used as a tool against corruption seems mixed.

2 The model

We consider two versions of an infinite horizon game between an electorate and a sequence of politicians: the no commitment game, in Section 5, and the commitment game, in which the electorate can commit to a policy at the beginning of the game, in Section 6.

In both cases, all players have a common discount factor δ, with $\delta \in (0, 1)$. We use average present value. There are two types of politician: honest ($h = 1$) and corrupt ($h = 0$). Time is discrete: $t = 1, 2, ...$

In the no commitment case, the electorate and politician play a stage game t as follows:

- At the beginning of period t the politician in office is either honest ($h_t = 1$) or corrupt ($h_t = 0$), and has a prior *reputation* or public belief of being honest of η_t, with $\eta_t \in [0, 1]$.
- Nature *reveals* the politician's type. The prior η_t is correspondingly updated to either 0 (if corrupt) or 1 (if honest). The updated prior, or interim reputation, is denoted by $\underline{\eta}_t$, and is equal to the actual type h_t.
- Nature also chooses a state u_t from a fixed pdf $f(u_t)$, with cdf $F(u_t)$. This represents the opportunity to receive a bribe where u_t is the size of the bribe. We assume that f has full support over $[0, 1]$ - a normalization. The state is only observed by the politician in office.
- The politician in office, having observed u_t, chooses an action $a_t \in \{0, 1\}$, 0 being the honest action, and 1 the corrupt action - taking a bribe. The honest type always chooses the honest action; the corrupt type is a strategic type.
- When choosing the corrupt action, the politician knows that there is a probability κ of that fact being exposed to the public. The indicator function k_t takes on the value 1 if the corrupt action is exposed at t, and 0 otherwise. Thus, $\kappa \equiv E[k_t]$. Notice that only the acceptance of the bribe can be observed, not the size.
- The electorate chooses a probability of keeping the incumbent β_t - a mixed strategy. Its realization is either keeping the politician in office ($b_t = 1$) or replacing her ($b_t = 0$). Thus, $\beta_t = E[b_t]$. The choice is made with the information at hand: the incumbent's type h_t, and the realization k_t of whether the bribe is observed.
- Nature chooses the type for next period's politician, h_{t+1}. If the incumbent was re-elected, a corrupt incumbent remains corrupt, while an honest one has a probability λ of remaining honest. If instead last period's politician was not re-elected, or if the game is about to start, Nature fills-in the position with a randomly drawn politician from a given pool. A new politician is honest with probability η_0. We assume that $\lambda > \eta_0$; we will show later that this condition guarantees that the known honest politician is preferred to a new draw from the moral roulette.

The stage game thus ends with re-election or replacement. A voted out politician can never be a candidate again. The timeline is shown in Figure 1.

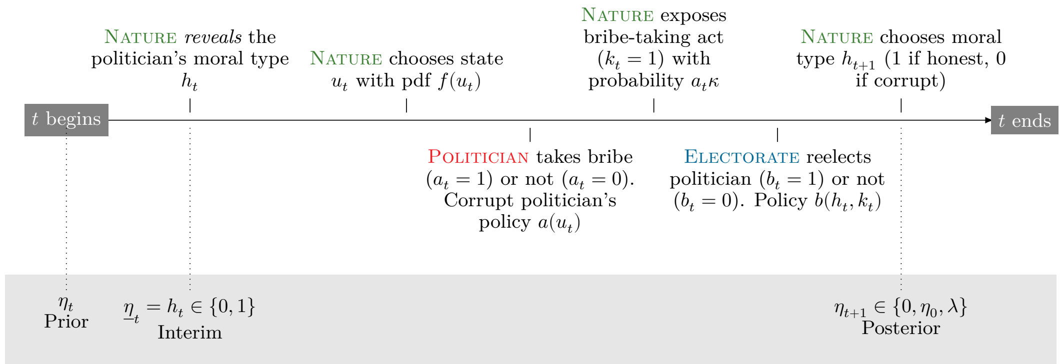


Figure 1: Timeline for the stage game

Payoffs are as follows:

- The politician's period payoff to action $a_t = 0$ is normalized to 0; the period payoff to the corrupt action is state-dependent: $u_t \geq 0$.
- If the honest action is taken the electorate receives zero. The corrupt action costs it σu_t, where $\sigma \geq 1$. If $\sigma = 1$, the stage game between the corrupt politician and the electorate is a zero-sum one. A parameter $\sigma > 1$ means that corrupt actions are not mere transfers, but instead they destroy value - they are inefficient. For instance, this would be the case if the politician initiates an unnecessary public infrastructure project so that her party comrades are awarded the contract. Hence $\sigma - 1$ can be interpreted as a measure of the inefficiency entailed in corruption.
- The replacement action $(b_t = 0)$ costs $c > 0$ to the electorate and nothing to the politician.
- The payoff to the politician that is not in office is null.

In this game there are not proper subgames as the politician may know the past draws of u_t while the electorate does not. However, these past payoffs are irrelevant for the continuation game so we can define a version of Markov perfect equilibrium in which we treat each period as the beginning of one of two types of games depending only on whether the politician is known to be corrupt or not. In the latter case, in the first period of the continuation game, the honest politician rejects the bribe. In the former case denote by $a(u_t)$ the decision of the corrupt politician whether or not to accept the bribe based on how large it is. In both cases denote the first period strategy of the electorate by $\beta(h_t, k_t)$. Notice the Markov assumption that both of these strategies depend on the past only on whether the politician is known to be honest at the beginning of the period.

In the no commitment case our equilibrium concept is that these strategies should form a Nash equilibrium of the game beginning in period t. In the commitment case $\beta(h_t, k_t)$ is chosen once and for all at the beginning of the first period and the politician is assumed to respond optimally.

3 Main Result

Our main result characterizes equilibrium in all cases. First we define the *posterior* reputation at t, which will also be next period's *prior*, as η_0 if the politician were replaced, 0 if the dishonest politician were kept, and λ if the honest politician were kept. That is, the posterior is

$$\eta_{t+1} = \begin{cases} \eta_0 & \text{if } b_t = 0, \\ 0 & \text{if } b_t = 1 \wedge h_t = 0, \text{ and} \\ \lambda & \text{if } b_t = 1 \wedge h_t = 1. \end{cases} \tag{1}$$

Theorem 1. *Regardless of whether commitment is possible or not the electorate never replaces an honest politician.*

If the electorate cannot commit, in a strict Markov equilibrium the corrupt politician takes all bribes. There is a unique cutoff c_1 such that regardless of whether a bribe is observed a corrupt politician is replaced if $c < c_1$ and re-elected if $c > c_1$.

If the electorate can commit to pure strategies then there are unique cutoffs c_0, c_2 such that the optimal policy for the electorate is given by the probability of re-electing a dishonest politician in the absence of an observed bribes $\hat{\beta}_0$ and the probability when a bribe is observed $\hat{\beta}_1$ and

$$(\hat{\beta}_0, \hat{\beta}_1) = \begin{cases} (0,0) & \textit{if } c < c_0, \\ (1,0) & \textit{if } c_0 < c < c_2, \textit{ and} \\ (1,1) & \textit{if } c > c_2. \end{cases} \tag{2}$$

In the first and last case the politician accepts all bribes while in the intermediate case there is a unique γ^* such that the bribe is accepted if and only if $u_t \geq \gamma^*$. If the *electorate could commit to mixed strategies, for some parameter values it would do so in order to avoid paying the replacement cost too often.*

The unique equilibrium values of c_1, c_0, c_2, γ^* *are computed below.*

The first part (no commitment) of the Theorem is proven as Corollary 1 below, while the second part (commitment) follows from Propositions 3 and 1 below.

The first part (no commitment) of the Theorem as promised shows that in this case corruption is endemic and either politicians are never replaced or always replaced depending on how the cost of replacement compares with the chances of getting another corrupt politician.

The second part (commitment) for low or high replacement cost is similar to the no commitment case, but has the crucial additional intermediate case in which politicians are replaced exactly when they are caught taking bribes, and as a result take only lucrative bribes.

Moreover, under commitment we demonstrate (Proposition 6) that advancements in monitoring technology could potentially diminish the electorate's welfare when exclusively pure strategies are available. This phenomenon occurs because, in scenarios where the technology is "excessively effective," voters employing the incentive strategy may terminate the politician's tenure with undue frequency. The availability of mixed strategies empowers voters to emulate inferior information technologies, thereby averting a decline in welfare attributable to the technology.

4 The corrupt politician's problem

The honest politician is a behavioral type that always passes up bribing opportunities. Here we examine the best response of the corrupt politician to a Markov policy of the electorate and show that it is described by a cutoff function in which only lucrative bribes as measured by u_t the value of the bribe are accepted.

Observe that the probability the electorate re-elects a corrupt politician upon exposure and upon non-exposure, by the Markov property must be constant over time, so may be denoted respectively by β_1 and β_0. Hence, the corrupt politician's decision problem is to choose a_t, that is whether to accept the corruption opportunity u_t, as described by the following Bellman equation:

$$v(u_t) = \max_{a_t \in \{0,1\}} \left\{ a_t \Big((1-\delta)u_t + \delta \Big[\kappa\beta_1 V + (1-\kappa)\beta_0 V \Big] \Big) + (1-a_t)\delta\beta_0 V \right\}. \tag{3}$$

where recall that $\kappa \equiv E[k_t]$ is the expected probability of exposure and

$$V \equiv E[v(u)] \tag{4}$$

is the expected continuation value, where the expectation is taken with respect to u.

Eq. (3) shows that the consequences of taking the bribe ($a_t = 1$) are getting u_t in the present, and then, with a probability $\kappa\beta_1$ being exposed and re-elected, or instead, not being exposed and still be re-elected with probability $(1-\kappa)\beta_0$. Not taking the bribe gives nothing in the present, and leads to reelection with probability β_0. Being replaced or re-elected after each circumstance are the electorate's choice (β_0, β_1).

Our basic result concerning the politician's behavior is this:

Lemma 1 (Politician's cutoff policy)**.** *In a Markov equilibrium the corrupt politician must use a cutoff rule: accept the offer exactly in those states in which $u_t \geq \hat{\gamma}$, that is,*

$$\hat{a}(u_t) = \begin{cases} 1 & \text{if } u_t \geq \hat{\gamma}, \ \textit{and} \\ 0 & \textit{otherwise}, \end{cases} \tag{5}$$

where

$$\hat{\gamma} \equiv \kappa \frac{\delta}{1-\delta}(\beta_0 - \beta_1)V. \tag{6}$$

Proof. In the Appendix. □

The politician weights the loot u_t against the chances of getting caught κ. Her choice is to take the high-gain opportunities, and pass up the low-gain ones. The more attractive corruption opportunities are naturally those that offer the largest payoffs.

For given V, the cutoff $\hat{\gamma}$ is increasing in the discount factor δ: the more patient she is, the more she is willing to let go in the short run so as to keep the opportunity of taking larger bribes in the future.

On the other hand, the cutoff is affected by the electorate's strategy. In order for the cutoff to be positive, it is necessary that the electorate rewards good behavior (or rather, the inexistence of evidence of bad behavior); for if the electorate's response were the same once the corrupt action were exposed than otherwise, the corrupt politician would simply steal at every opportunity. Formally, $\beta_0 = \beta_1$ implies $\hat{\gamma} = 0$, as can be seen in Eq. (6).

Finally, the cutoff is also increasing in the continuation value V: the larger the present value of the stream of prizes, the more willing to let go the small and dangerous (i.e., likely-to-be-exposed) bribing opportunities.

Observe that a given cutoff γ determines an ex ante expected loot $\nu \equiv E[ua(u)]$, that is,

$$\nu(\gamma) = \int_{\gamma}^{1} u f(u) \mathrm{d}u. \tag{7}$$

This function is monotonically decreasing in the cutoff γ.

Observe that when $\gamma = 0$ the corrupt politician takes all bribes and the expected loot reaches its maximum value, namely $\nu = E[u]$. At the other extreme, a unitary cutoff would mean $\nu = 0$ because $a(u)$ would be 0 with probability 1 (i.e., almost surely –a.e.).

Proposition 1 below characterizes the corrupt politician's choice of γ. Recall that the cdf of the bribe u is denoted by $F(u)$.

Proposition 1 (Corrupt politician's best reply)**.** *If $\beta_0 - \beta_1 \leq 0$ the unique optimum is $\hat{\gamma} = 0$. If $\beta_0 - \beta_1 > 0$, for each pair (β_0, β_1) there is a unique optimum $\hat{\gamma}$ defined by*

$$\hat{\gamma} = \frac{\kappa\delta(\beta_0 - \beta_1)}{1 - \delta B(\hat{\gamma})} \nu(\hat{\gamma}) \tag{8}$$

where $B(\gamma) \equiv (1 - F(\gamma))(\kappa\beta_1 + (1 - \kappa)\beta_0) + F(\gamma)\beta_0 = \beta_0 - (1 - F(\gamma))\kappa(\beta_0 - \beta_1)$ is the expected probability of reelection of a corrupt politician. Moreover, the optimum satisfies

$$\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\beta_0} > 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\beta_1} < 0.$$

Proof. Indeed, from Lemma 1 the best reply of the politician is taking the bribes if u is greater than a given γ. Computing the expectation of Eq. (3) under the policy $a(u) = \mathbf{1}_{\{u \geq \gamma\}}$ (where $\mathbf{1}_A$ is the indicator function of the set A) we get

$$V = \int_{\gamma}^{1} \Big((1 - \delta) u_t + \delta \Big[\kappa\beta_1 V + (1 - \kappa)\beta_0 V \Big] \Big) f(u) \mathrm{d}u + \int_{0}^{\gamma} \delta\beta_0 V f(u) \mathrm{d}u.$$

Evaluating the integrals and collecting V we get

$$V(1-\delta\Big[\kappa\beta_1+(1-\kappa)\beta_0\Big](1-F(\gamma))-\delta\beta_0F(\gamma))=(1-\delta)\nu(\gamma).$$

Rearranging, we obtain

$$V=\frac{1-\delta}{1-\delta B(\gamma)}\nu(\gamma). \tag{9}$$

Replacing in (6) the cutoff becomes

$$\hat{\gamma}=\frac{\kappa\delta\left(\beta_0-\beta_1\right)}{1-\delta B\left(\hat{\gamma}\right)}\nu\left(\hat{\gamma}\right). \tag{10}$$

Let

$$g\left(\gamma\right)\equiv\frac{\kappa\delta\left(\beta_0-\beta_1\right)}{1-\delta B(\gamma)}\nu(\gamma) \tag{11}$$

be the right hand side of Eq. (10), so that $\hat{\gamma}$ is a fixed point of g. It can be seen that $g'\left(\gamma\right)\gtreqless 0 \iff \gamma\lesseqgtr g(\gamma)$ since

$$g'\left(\gamma\right)=\frac{\kappa\delta\left(\beta_0-\beta_1\right)f\left(\gamma\right)}{\left(1-\delta B\left(\gamma\right)\right)^2}\left(-\gamma\left(1-\delta B\left(\gamma\right)\right)+\kappa\delta\left(\beta_0-\beta_1\right)\nu(\gamma)\right).$$

Also, $g'\left(\hat{\gamma}\right)=0$, implying that g has a unique fixed point, namely $\hat{\gamma}$. Let θ be any parameter of g. The fact that $g'\left(\hat{\gamma}\right)=0$ also implies that

$$\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\theta}=\frac{\partial g}{\partial\theta} \tag{12}$$

because $\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\theta}=\frac{\partial g}{\partial\theta}+g'\left(\hat{\gamma}\right)\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\theta}$. Differentiating g with respect to each beta in turn, we get

$$\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\beta_1}=-\delta\kappa\nu\left(\gamma\right)\frac{1-\delta\beta_0}{(1-\delta B)^2}<0 \tag{13}$$

and

$$\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\beta_0}=\delta\kappa\nu\left(\gamma\right)\frac{1-\delta\beta_1}{(1-\delta B)^2}>0, \tag{14}$$

as asserted. □

Re-election serving as a reward for corrupt politicians creates a perverse incentive. A higher likelihood of re-election following corruption exposure compared to non-exposure encourages the politician to maximize bribe-taking. Conversely, only when

$\beta_0 > \beta_1$ does a corrupt politician exhibit restraint. Moreover, the cutoff for taking bribes is increasing in the probability of reelection after not being exposed (β_0) and decreasing in the probability of reelection after being exposed (β_1). As a consequence, the strongest incentives to avoid corruption are provided by the strategy $\beta_1 = 0$ and $\beta_0 = 1$ - which will henceforth be referred to as "the maximal incentive strategy," or simply "incentive strategy" for short. Under this strategy, the highest equilibrium cutoff γ^* is obtained, and is the solution to

$$\gamma^* = \frac{\delta\kappa}{1 - \delta\left(1 - \kappa(1 - F(\gamma^*))\right)}\nu(\gamma^*). \tag{15}$$

Clearly, $0 < \gamma^*$, and generic strategies β result in $\hat{\gamma} \in [0, \gamma^*]$.

We now turn to the electorate's choice. In this case commitment matters a great deal to the equilibrium strategy of the electorate, so we study the cases with and without commitment separately.

5 The game without commitment

In this section we explore the question of whether once it is known that a politician is corrupt it is still optimal to give her incentives not to take bribes by threatening her with destitution - and thereby removing the possibility of future bribes - when bribes are observed. The answer, as it turns out, is negative.

Recall that σu is the cost to the electorate of the bribe u. Then the electorate's value function is given by

$$\begin{aligned} w(h_t, k_t) = \max_{\beta_{k_t} \in [0,1]} \Big\{ &(1-\delta)\Big(-(1-h_t)\sigma\big(k_t\nu_1 + (1-k_t)\nu_0\big) - (1-h_t)\left(1-\rho_t\right)c\Big) + \\ &\delta\Big(h_t W(\lambda) + (1-h_t)\left(\rho_t W(0) + (1-\rho_t)\, W(\eta_0)\right)\Big)\Big\}, \end{aligned} \tag{16}$$

where $\rho_t \equiv k_t\beta_1 + (1-k_t)\beta_0$ is the probability of reelecting a corrupt politician given $k_t \in \{0, 1\}$, $\nu_1 \equiv E[ua(u)|k = 1]$ is the expected size of the bribe given that a bribe taking act was exposed, $\nu_0 \equiv E[ua(u)|k = 0]$ is the expected size of the bribe given that a bribe taking act wasn't exposed, $\nu \equiv \left(1 - \kappa(1 - F(\gamma))\right)\nu_0 + \kappa(1 - F(\gamma))\nu_1 = E[ua(u)]$ is the expected size of the bribe taken by a corrupt politician, and

$$W(\eta) \equiv E[w(h, k)] \tag{17}$$

is the expected continuation payoff of a stage that begins with a prior reputation η. Eq. (16) says that the electorate's expected flow payoff (whose value is discounted by $(1-\delta)$) when facing a corrupt politician is the expected bribe amplified by the factor σ, minus the cost of replacing the politician c if that is the choice ($\rho = 0$); plus the expected continuation payoff, multiplied by the factor δ. The continuation payoff is the expected value of either maintaining the current politician - i.e., starting next period with a politician who is honest with probability λ of honest today and 0 if corrupt today - or replacing her with another who will be honest with probability η_0.

Proposition 2 (Electorate's policy without commitment)**.** *Consider a fixed politician's policy $a(u) = \mathbf{1}_{\{u \geq \gamma\}}$ generating an expected bribe value of $\nu(\gamma)$. Then, the electorate's optimal policy involves reelecting the honest politician, while when facing a dishonest one, reelect her if it is too expensive to replace her and replace her otherwise, irrespective of whether she was caught taking a bribe or not. Formally,*

$$(\hat{\beta}_0, \hat{\beta}_1) = \begin{cases} (1,1) & \textit{if } c > \hat{c}\,(\gamma) \\ (0,0) & \textit{if } c < \hat{c}\,(\gamma) \end{cases} \tag{18}$$

where

$$\hat{c}\,(\gamma) \equiv \frac{\delta \eta_0}{1-\delta\lambda} \sigma \nu\,(\gamma)\,. \tag{19}$$

Proof. In the Appendix.Except for the knife-edge case of indifference, the optimal policy is unconditional in the sense that it considers doing the same regardless of the corrupt act having being exposed or not ($\beta_0 = \beta_1$). Indeed, knowing that the politician took a bribe, or not knowing it, affects the expected payoff for the stage game but not the continuation payoff. This is to say, exposure of the corrupt act is inconsequential to the expectation of future payoffs, and therefore the rational electorate doesn't take this event into consideration when deciding whether to reelect the politician or not. Consequently, the corrupt politicians do not have incentives to not take bribes. □

Corollary 1. *If the electorate cannot commit, in a strict Markov equilibrium the corrupt politician takes all bribes ($\hat{\gamma} = 0$), and $\hat{c}\,(0) = c_1$, with*

$$c_1 \equiv \frac{\delta \eta_0}{1-\delta\lambda} \sigma \nu\,(0)\,. \tag{20}$$

Proof. Simply note that Eq. (6) evaluates to 0 if $\beta_0 = \beta_1$. Consequently, $a(u) = 1$

for all u, and $\nu = E[ua(u, \kappa)] = \nu(0)$. □

Thus, in a strict equilibrium $\hat{a}(u) = 1$, $\hat{\nu} = E[u] = \nu(0)$, and the politician is never replaced if the cost is higher than the cutoff $\hat{c}$ while is always replaced if the cost is smaller than that.

This cutoff cost is proportional to η_0. If, on the extreme, the expectations are so low that the electorate believes all politicians to be corrupt ($\eta_0 = 0$), the situation is hopeless and the smallest replacement cost induces the electorate to always reelect the known-corrupt politician. This results in not punishing corrupt politicians even when they are caught. Indeed, Pavão (2018) finds that when corruption is pervasive voters indeed tend to overlook it.

The factors that increase this cutoff include the discount factor δ (the more patient the electorate, the higher the standard for reelection); the probability that the honest remains honest λ (the higher it is, the higher the cutoff); and the expected cost of corruption, which is the product of the expected loot times its damage factor, $\sigma\nu(0)$ (the more damaging corruption is, the higher the standard for reelecting the politician).

It is important to note that the variables δ, η_0, σ, and $\nu(0)$ enter the cutoff cost in a multiplicative fashion, indicating a strong form of complementarity. Indeed, none of them affects the cutoff $\hat{c}$ if any of the others is null; conversely, their effect on the cutoff is higher the higher the value of the rest of the variables. For instance, the better the pool of alternative candidates η_0, the larger the effect of the expected bribes on the cutoff.

So far we have referred only to strict equilibria. It turns out that there are also mixed strategy equilibria (described in Appendix B) that involve a cost that makes the electorate indifferent among all strategies. These equilibria face the familiar criticisms of mixed strategies, namely, that they are not robust (Fudenberg, Kreps, and Levine 1988) and that the electorate would be acting for no good reason, among others. Moreover, these equilibria are not generic in the parameter space.

Things are different if the electorate may commit to act differently upon exposure than upon non exposure, as this would incentivize the politician to behave honestly when the realization of u - the loot - is small enough. Doing so requires commitment because the electorate would need to keep the politician when it would prefer to get rid of her, or to destitute her when it would rather reelect her. This is the case we analyze next. It turns out that the policy of promising to reelect the corrupt politician if she was not caught taking bribes and depose her otherwise, is optimal for intermediate values of the reelection cost c.

6 The game with pure commitment

We assume now that the electorate can commit at the beginning of time to a pure[1] strategy (β_0, β_1) when facing corrupt politicians, and implement it from then onward. For instance, $\beta_1 = 0$ – not reelecting a politician that was caught taking a bribe – could be achieved by a constitution that makes the higher office incompatible with a corruption sentence. We discuss other forms of commitment later.

The following proposition shows that the incentive strategy $(\beta_0, \beta_1) = (1, 0)$, namely, replacing the politician if and only if she was caught taking a bribe, is the best policy the electorate can choose, in a given parameter region. This policy induces a cutoff γ^*, as defined by Eq. (15). This induces the smallest equilibrium probability of corrupt actions in any Markov equilibria.

Proposition 3 (Electorate's policy with pure commitment)**.** *There exist critical values*

$$c_0 \equiv \frac{\sigma}{1-\kappa\left(1-F\left(\gamma^*\right)\right)}\left(\nu\left(\gamma^*\right)\left(1+\frac{\delta\eta_0}{1-\delta\lambda}\right)-\nu\left(0\right)\left(1+\frac{\delta\eta_0}{1-\delta\lambda}\kappa\left(1-F\left(\gamma^*\right)\right)\right)\right) \tag{21a}$$

$$c_2 \equiv \frac{\sigma\delta\eta_0}{1-\delta\lambda}\nu(0)+\frac{\sigma}{\kappa\left(1-F\left(\gamma^*\right)\right)}\left(\nu\left(0\right)-\nu\left(\gamma^*\right)\right), \tag{21b}$$

such that the optimal policy is

$$(\hat{\beta}_0, \hat{\beta}_1) = \begin{cases} (0,0) & \textit{if } c < c_0, \\ (1,0) & \textit{if } c_0 < c < c_2, \textit{ and} \\ (1,1) & \textit{if } c > c_2. \end{cases} \tag{22}$$

Proof. The analysis is similar to that of Proposition 2 except that now the electorate takes into account the effect of its policy on the corrupt politician's behavior as per Proposition 1.

[1]We consider mixed strategies in the next section

Starting from a given electorate's strategy (β_0, β_1) we may compute an electorate's value function from Eq. (17) as

$$P(\beta_0,\beta_1,\eta) = (1-\delta)\Big(-(1-\eta)\left(\sigma\nu(\gamma)+(1-B)\,c\right)\Big) \\ +\delta\Big(\eta P(\beta_0,\beta_1,\lambda)+(1-\eta)\left(BP\left(\beta_0,\beta_1,0\right)+(1-B)P(\beta_0,\beta_1,\eta_0)\right)\Big),$$

where $P(\beta_0, \beta_1, \eta)$ is the lifetime discounted average payoff from following a fixed strategy (β_0, β_1) with an initial reputation η, $B(\gamma, \beta_0, \beta_1) \equiv (1 - F(\gamma))(\kappa\beta_1 + (1-\kappa)\beta_0)+$ $F(\gamma)\beta_0$ or equivalently

$$B(\gamma,\beta_0,\beta_1) = \beta_0 - (1-F(\gamma))\,\kappa\,(\beta_0-\beta_1) \tag{23}$$

is the expected probability of reelection of a corrupt politician, and γ is understood as the equilibrium best response $\hat{\gamma}(\beta_0, \beta_1)$. Maximizing $P(\beta_0, \beta_1, \eta)$ with respect to (β_0, β_1) would yield the value function $W(\eta)$. Evaluating $P(\beta_0, \beta_1, \eta)$ at $\eta = 0$, $\eta = \lambda$, and $\eta = \eta_0$, and solving the resulting system of equations, we get

$$P(\beta_0,\beta_1,\eta) = \frac{(\sigma\nu(\gamma)+(1-B)c)}{1-\delta\lambda+\delta\eta_0(1-B)}(1-\delta)\left(-\frac{1-\delta\lambda}{1-\delta}+\eta\right). \tag{24}$$

Let

$$T(\eta) \equiv (1-\delta)\left(-\frac{1-\delta\lambda}{1-\delta}+\eta\right) < 0$$

be the second term, so that

$$P(\beta_0,\beta_1,\eta) = S(\beta_0,\beta_1)T(\eta)$$

where

$$S(\beta_0,\beta_1) \equiv \frac{\sigma\nu(\hat{\gamma}(\beta_0,\beta_1)) + (1-B(\hat{\gamma}(\beta_0,\beta_1),\beta_0,\beta_1))c}{1-\delta\lambda+\delta\eta_0(1-B(\hat{\gamma}(\beta_0,\beta_1),\beta_0,\beta_1))} \tag{25}$$

is the slope of the value function. The maximization of $P(\beta_0, \beta_1, \eta)$ requires minimizing $S(\beta_0, \beta_1)$.

Evaluating, we see that $S(1, 0) < S(0, 0)$ obtains if and only if

$$c > \frac{\sigma}{1-\kappa(1-F(\gamma^*))}\left(\nu(\gamma^*)\left(1+\frac{\delta\eta_0}{1-\delta\lambda}\right)-\nu(0)\left(1+\frac{\delta\eta_0}{1-\delta\lambda}\kappa(1-F(\gamma^*))\right)\right) \equiv c_0. \tag{26}$$

Similarly, $S(1,0) < S(1,1)$ obtains if and only if

$$c < \frac{\sigma\delta\eta_0}{1-\delta\lambda}\nu(0) + \frac{\sigma}{\kappa\left(1-F\left(\gamma^*\right)\right)}\left(\nu\left(0\right) - \nu\left(\gamma^*\right)\right) \equiv c_2. \tag{27}$$

By the same token, $S(0,0) < S(1,1)$ if and only if $c > c_1$. It is easy to verify that $c_0 < c_1$ and $c_1 < c_2$ if and only if $\gamma^* > 0$, which indeed holds true. The optimal policy is thus

$$(\hat{\beta}_0, \hat{\beta}_1) = \begin{cases} (0,0) & \text{if } c < c_0, \\ (1,0) & \text{if } c_0 < c < c_2, \\ (1,1) & \text{if } c_2 < c, \end{cases} \tag{28}$$

as asserted. □

Observe first that under commitment it is still optimal to never reelect the corrupt politician if the cost of replacement is low enough (namely, $c < c_0$), as is also still optimal to reelect her regardless of being caught taking bribes or not if the cost of replacing her is high enough (namely, $c > c_2$). What changes from the no-commitment case is that at intermediate replacement cost values (namely, $c \in (c_0, c_2)$), it is now optimal to reelect the politician in office if she is not caught taking bribes, and replace her whenever she is caught doing so. The benefit of such policy is inducing the dishonest politician to forego bribe-taking opportunities in which the bribes are small, motivated by the chances of being in office when a better opportunity presents itself. In other words, the benefit to the electorate is inducing a higher cutoff for taking bribes γ^*.

The proposition below characterizes the parameter region in which it is optimal to commit to this new policy.

Proposition 4 (The pure commitment region)**.** *Under the maintained assumptions that $\delta > 0$ and $\lambda > \eta_0$,*

i. the interval (c_0, c_2) is nonempty, while

$$\lim_{\delta\to 0} c_0 = \lim_{\delta\to 0} c_2 = 0;$$

ii. Let

$$c_1^* \equiv \frac{\sigma\delta\eta_0}{1-\delta\lambda}\nu\left(\gamma^*\right), \tag{29}$$

then $c_1^* < c_1$ and c_1^* is in the interior of (c_0, c_2), separating the regions (c_0, c_1^*), in which the electorate would rather replace the corrupt politician that has not been caught, and (c_1^*, c_2), where the electorate would rather reelect the corrupt politician that has been caught;

iii. the three cutoffs c_0, c_1^*, and c_2, are monotonically increasing in η_0 and λ, and are proportional to σ.

Proof. In the Appendix.It should be noted that c_0 could be negative, meaning that there could be no positive cost levels at which replacing the corrupt politician regardless of whether she was caught or not is optimal. Trivially, this is the case, for instance, when $\eta_0 = 0$: if a replacing politician is sure to also be corrupt, there is no benefit from replacing the one in office. In this case, $c_2 > 0$, meaning that for not-too-high cost levels, the optimal policy is indeed to reelect known corrupt politicians as long as they are not caught taking bribes. □

It also should be noted that c_2 could be very large; $\sigma\left(\nu\left(0\right) - \nu\left(\gamma^*\right)\right)$ is the expected period gain to the electorate from inducing the corrupt politician not to take all bribes, a strictly positive quantity, $\sigma > 1$ being a measure of the inefficiency entailed in corruption. In the same vein, the inverse of the equilibrium probability of being caught could itself be a large multiplier.

Corollary 2. *If the electorate can commit only to pure strategies, for intermediate costs of replacement in a strict Markov equilibrium the electorate commits to replace the corrupt politicians if and only if they are caught taking bribes, and the corrupt politician refrains from taking bribes below γ^*. Indeed,*

$$\left(\hat{\gamma}, \hat{\beta}_0, \hat{\beta}_1\right) = \begin{cases} (0,0,0) & \textit{if } c < c_0, \\ (\gamma^*, 1, 0) & \textit{if } c_0 < c < c_2, \\ (0,1,1) & \textit{if } c_2 < c. \end{cases}$$

There could be mixed strict equilibria as well; we discuss them in the next Section.

The discount factor plays an important role in our model. Should players not care about the future ($\delta = 0$), there would be no interest on the electorate in modifying current behavior to induce any response on the part of future politicians; this is reflected in the collapsing interval of incentives. Nor there would be an interest on the part of the corrupt politician to forego bribes in order to secure her future reelection ($\lim_{\delta\to0}\hat{\gamma}=0$, implying $\lim_{\delta\to0}\hat{\nu}=\nu(0)$). On the other hand, the more the players care about the future, the larger the commitment region becomes. Our next result concerns the effect the players' patience on equilibrium behavior under pure commitment.

Proposition 5 (The effect of patience on the corrupt politician's equilibrium behavior)**.** *Suppose $c\in(c_0,c_2)$. In a strict equilibrium under pure commitment, the corrupt politician's behavior satisfies*

$$\frac{\partial\gamma^*}{\partial\delta}>0.$$

Moreover,

$$\lim_{\delta\to0}\gamma^*=0 \quad \textit{and} \quad \lim_{\delta\to1}\gamma^*=1.$$

Proof. In the Appendix. □

Proposition 5 says that the more patient the corrupt politician is, the more stringent the criterion for accepting bribes she will adopt. As a consequence, she will take fewer bribes, and be exposed less often, and obtain a smaller per-period expected loot. All this, in exchange for a longer expected term in office. In the other direction, this says that the politician will not wait for the highest bribes, but instead will accept increasingly more bribes the less impatient she is.

We conclude this section by showing that a rise in information accuracy may lead to a decline in the welfare of the electorate.

Proposition 6 (Improvements in monitoring technology are not always welcome)**.** *In a strict equilibrium under pure commitment improvements in the informational content of the signal (κ) may reduce the electorate's welfare.*

Proof. The proof is by example. Consider a uniform distribution F over u. The parameters are set to $(c,\delta,\eta_0,\lambda,\sigma)=(1.5,0.95,0.2,0.8,1)$. Under these assumptions, $c_1=\frac{19}{48}$. Figure 2 shows value functions $P(\beta_0,\beta_1,\eta)$ for the incentive policy $(1,0)$ and strategy $(1,1)$ as well. Observe that $P(1,0,0)$ is increasing in the A-C region and decreasing elsewhere. On the other hand, $P(1,0,0)$ is larger than $P(1,1,0)$ - and therefore optimal,and equal to the value function - in the B-D region and smaller elsewhere. From C to D then the electorate's welfare is indeed decreasing, as asserted. As the information technology (represented by κ) improves, incentives are strenghtened and the corrupt politician indeed becomes more selective (higher $\hat{\gamma}$). However, gains from incentives are countered by increased costs from getting

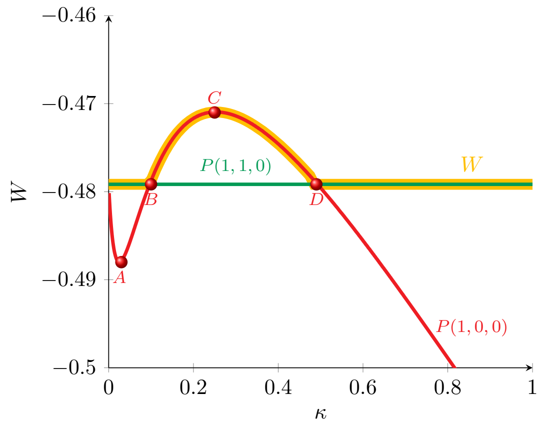


Figure 2: A non monotonic value function.

This numerical example is generated by a uniform distribution F over u. The parameters are set to $(c, \delta, \eta_0, \lambda, \sigma) = (1.5, 0.95, 0.2, 0.8, 1)$. Under these assumptions, $c_1 = \frac{19}{48}$. The picture shows the value functions $P(\beta_0, \beta_1, \eta)$ for strategies with $\beta_0 = 1$ and $\eta = 0$ (i.e., the electorate facing a known corrupt politician) for the two values of β_1. The optimal strategy is thus $(1, 1)$ for the extreme values of κ and $(1, 0)$ for intermediate ones.

rid of politicians more often. The latter effect dominates between C and D. Beyond D, providing incentives becomes so expensive that is ceases to be worth it and the electorate goes back to the always reelecting policy. □

By the end of next section we go back to this result but then considering that the electorate could use mixed strategies.

7 Mixed commitment

This Section considers the choice when all mixed strategies $(\beta_0, \beta_1) \in [0, 1]^2$ are available. Through its commitment to a mixed strategy the electorate induces the corrupt politician to select a given cutoff γ and also induces a probability of reelection B. The results of this section are summarized in the following proposition:

Proposition 7 (Electorate's policy with mixed commitment)**.** *The optimal strategy is either a pure one from the set* $\{(0, 0), (1, 0), (1, 1)\}$*, or a convex combination of* $\{(0, 0), (1, 0)\}$ *or of* $\{(1, 0), (1, 1)\}$*.*

We prove this proposition in different segments. We characterize first the combinations of (γ, B) that are feasible and secondly we study the preferences and the option that maximizes the electorate's utility.

The feasible set

When the electorate chooses an strategy $(\beta_0, \beta_1) \in [0,1]^2$, the corrupt politician's behavior gives rise to a pair (γ, B),the cutoff for accepting bribes and the probability of re-election defined implicitly by the equations (10) and (23), namely

$$\hat{\gamma} = \max\left\{0, \frac{\kappa\delta\left(\beta_0 - \beta_1\right)}{1 - \delta B\left(\hat{\gamma}\right)}\nu\left(\hat{\gamma}\right)\right\} \text{ and}$$
$$B\left(\hat{\gamma}\right) = \beta_0 - \left(1 - F\left(\hat{\gamma}\right)\right)\kappa\left(\beta_0 - \beta_1\right).$$

These equations define a mapping $(\gamma, B) = M(\beta_0, \beta_1)$. The feasible set of (γ, B) pairs, denoted by $\mathcal{F}(\kappa, \delta)$, is the image of operator, $\mathcal{F}(\kappa, \delta) = M\left([0,1]^2\right)$. Figure 3 plots an example in the plane (γ, B). It is bounded above by the image of the convex combinations of the strategies $(1,1)$ and $(1,0)$, i.e., they involve $\beta_0 = 1$, and

$$\frac{\partial B}{\partial \gamma} = \frac{\partial B/\partial \beta_1}{\partial \gamma/\partial \beta_1} = -\frac{1 - F(\gamma)}{\nu(\gamma)}\frac{1 - \delta B}{\delta(1-\delta)} < 0. \tag{30}$$

It is bounded below by the convex combinations of the strategies $(0,0)$ and $(1,0)$, i.e., they involve $\beta_1 = 0$ and

$$\frac{\partial B}{\partial \gamma} = \frac{\partial B/\partial \beta_0}{\partial \gamma/\partial \beta_0} = \frac{1 - \kappa(1 - F(\gamma))}{\nu(\gamma)}\frac{1 - \delta B}{\delta\kappa} > 0. \tag{31}$$

Finally, it is bounded to the left by all strategies that consider $\beta_0 \leq \beta_1$ which induce a $\hat{\gamma} = 0$.

The feasible set is increasing both in the probability that bribery is exposed κ and the degree of patience of politicians and the electorate δ. This is illustrated in Fig. 4 as way of example where the bribe opportunity distribution is uniform.

Lemma 2. *The feasible set is increasing in κ and δ, ie.,*

$$\kappa < \kappa' \implies \mathcal{F}(\kappa, \delta) \subset \mathcal{F}(\kappa', \delta) \quad \text{and} \quad \delta < \delta' \implies \mathcal{F}(\kappa, \delta) \subset \mathcal{F}(\kappa, \delta').$$

Proof. Let $(\gamma, B) \in \mathcal{F}(\kappa, \delta)$. If κ increased to κ' the same (γ, B) could be selected by

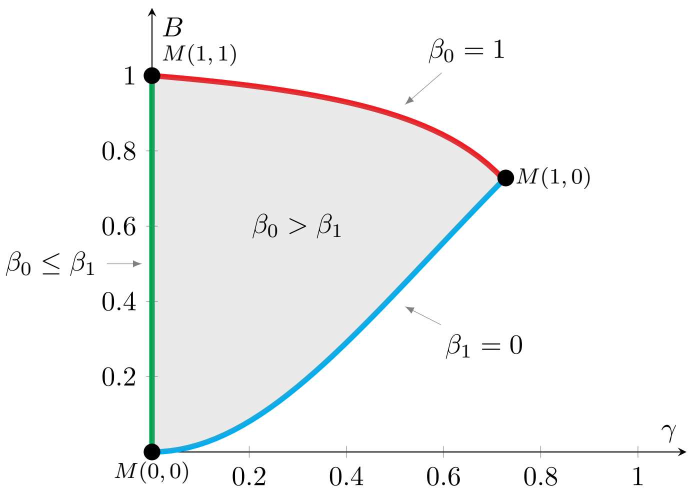
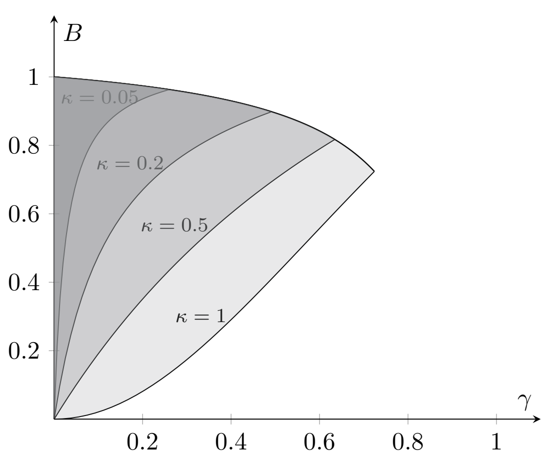
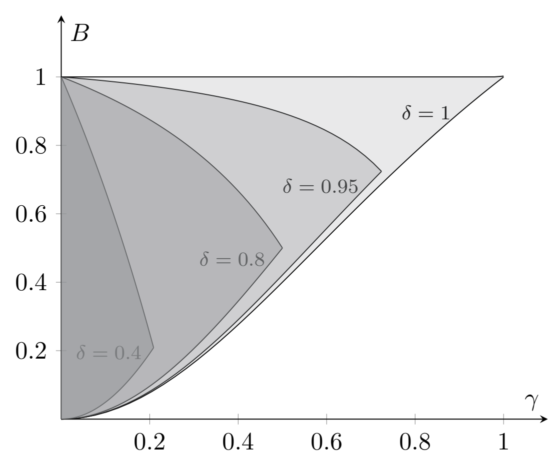


Figure 3: The feasible set

This numerical example is generated by a uniform distribution F over u and the parameters $(\kappa, \delta) = (1, 0.95)$.



(a) The effect of κ ($\delta = 0.95$).



(b) The effect of δ ($\kappa = 1$).

Figure 4: Comparative statics of the feasible set

This numerical example is generated by a uniform distribution F over u.

choosing $\beta_0' = \beta_0$ and $\beta_1' = \left(1 - \frac{\kappa}{\kappa'}\right)\beta_0 + \frac{\kappa}{\kappa'}\beta_1 \in [0,1]$ so that $(\beta_0' - \beta_1') = \frac{\kappa}{\kappa'}(\beta_0 - \beta_1)$

$$B' = \beta_0' - (1 - F(\hat{\gamma}))\kappa'(\beta_0' - \beta_1') = \beta_0 - (1 - F(\hat{\gamma}))\kappa'\frac{\kappa}{\kappa'}(\beta_0 - \beta_1) = B \text{ and}$$

$$\hat{\gamma}' = \max\left\{0, \frac{\kappa'\delta(\beta_0' - \beta_1')}{1 - \delta B'}\nu(\hat{\gamma}')\right\} = \max\left\{0, \frac{\kappa\delta(\beta_0 - \beta_1)}{1 - \delta B(\hat{\gamma})}\nu(\hat{\gamma})\right\} = \hat{\gamma}.$$

Then, $(\gamma, B) \in \mathcal{F}(\kappa, \delta) \implies (\gamma, B) \in \mathcal{F}(\kappa', \delta)$. As for δ, it doesn't enter the equation for B so it doesn't affect it; as for $\hat{\gamma}$,it suffices to choose $(\beta_0' - \beta_1') = \frac{\delta}{\delta'}(\beta_0 - \beta_1) < 1$ so as to maintain it. Therefore, $(\gamma, B) \in \mathcal{F}(\kappa, \delta) \implies (\gamma, B) \in \mathcal{F}(\kappa, \delta')$. □

The significance of this result is twofold. On the one hand, the quality of public auditing as represented by κ is crucial to the electorate. The larger it is, the more options the electorate has. Secondly, the same is true of the discount factors as incentives work better on the more patient corrupt politicians.

We now turn to study the electorate's preferences and optimality.

Preferences and optimality

Note that the utility of the electorate (Eq. (24)) depends on its strategy choice only through their influence on $\hat{\gamma}$ and $\hat{B}$, namely, the cutoff - which determines what bribes are taken - and the reelection probability - that determines both, the frequency at which the replacement cost c is paid and the chances of drawing a honest politician.

Observe that increasing γ is always utility-enhancing. Indeed, from Eq. (24) we have

$$\frac{\partial P}{\partial \gamma} = \frac{-\sigma\gamma f(\gamma)}{(1 - \delta\lambda + \delta\eta_0(1 - B))^2}T(\eta) > 0.$$

This implies that we need only consider mixed strategies of the form $(1, \beta_1)$ and $(\beta_0, 0)$, as only points in the right border of the feasible set could be optimal. The preference for B in contrast depends crucially on the replacement cost:

$$\begin{aligned}\frac{\partial P}{\partial B} &= \frac{-c(1 - \delta\lambda) + \delta\eta_0\sigma\nu(\gamma)}{(1 - \delta\lambda + \delta\eta_0(1 - B))^2}T(\eta) \gtreqless 0 \\ &\iff c \gtreqless \frac{\delta\eta_0\sigma}{(1 - \delta\lambda)}\nu(\gamma).\end{aligned}$$

When the replacement cost is too high, the electorate prefers to increase the probability of reelecting the corrupt politician B to save in replacement costs. When it is not that expensive, it prefers to decrease it to improve the chances of getting a honest politician. This agrees with Prop. 2: if all options available consider $\gamma = 0$, the electorate prefers $B = 1$ if $c > c_1$ and $B = 0$ if $c < c_1$. It also agrees with Prop. 3: if only the vertices are feasible in the choice set in (γ, B) the electorate will doubt between $(1, 1)$ and $(1, 0)$ if $c > c_1^*$ and between $(0, 0)$ and $(1, 0)$ if $c < c_1^*$. What needs to be considered is that attaining γ^* requires some sacrifice in B; the cutoffs c_0 and c_2 are the result of weighting these factors.

With mixed commitment all intermediate points also are available and the marginalist approach becomes adequate. The induced marginal rate of substitution between γ and B is the ratio of the previous derivatives, namely

$$\mathrm{MRS}(\gamma, B) = \frac{\partial P/\partial \gamma}{\partial P/\partial B} = \frac{-\sigma\gamma f(\gamma)}{-c(1-\delta\lambda) + \delta\eta_0\sigma\nu(\gamma)}. \tag{32}$$

Again, the MRS will be positive and utility increasing in the direction of higher γ and B if the cost of replacement c is high enough; otherwise, the MRS will be negative and the utility will increase in the direction of higher γ and smaller B. Observe also that the MRS depends only on γ, not on B, so that the indifference curve map in the plane (γ, B) is vertically parallel. In particular,

$$\mathrm{MRS}(0, 1) = \frac{-\sigma 0 f(0)}{-c(1-\delta\lambda) + \delta\eta_0\sigma\nu(0)} = 0 = \mathrm{MRS}(0, 0).$$

We observed before that $\frac{\partial P}{\partial \gamma} > 0$, i.e., the electorate welfare is always increasing in the corrupt politician's cutoff point. This implies that the optimum will occur at the right frontier of the feasible set $\mathcal{F}$. It could be at the top border - which is formed by the convex combinations of $\{(1,0), (1,1)\}$ - if $\frac{\partial P}{\partial B} > 0$, or at the bottom border - which is formed by the convex combinations of $\{(0,0), (1,0)\})$ - if $\frac{\partial P}{\partial B} < 0$.

This concludes the analysis leading to the statement of Prop. 7.

Notice that, there are parameter values at which all the optimal strategies are pure. Consider the following prominent case: $\kappa = \delta = 1$. The feasible set is the largest, and only the incentive strategy $(1, 0)$ is optimal for any cost $c \geq 0$. Indeed, when $\kappa = \delta = 1 = \beta_0$ M becomes $\hat{\gamma} = \max\left\{0, \frac{(1-\beta_1)}{1-B}\nu(\hat{\gamma})\right\}$ and $B = 1-(1-F(\hat{\gamma}))(1-\beta_1)$; replacing $(1-\beta_1)$ from the latter into the former equation, we see that $\hat{\gamma} = \frac{1}{(1-F(\hat{\gamma}))}\nu(\hat{\gamma})$

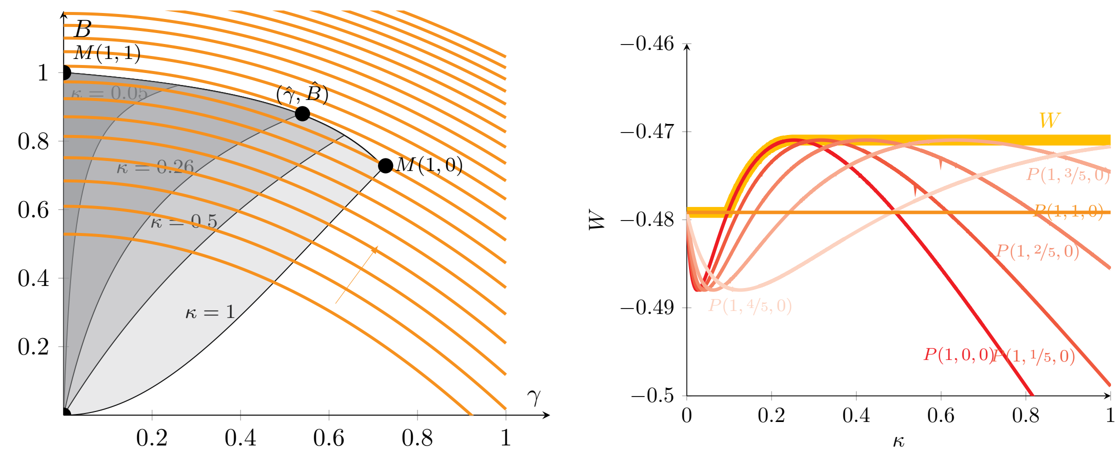
implying $\hat{\gamma} = 1$ as γ. becomes the expectation of u conditional on being larger than γ. At the same time, the MRS evaluated at $\gamma^* = 1$ is $MRS = \frac{-\sigma f(\gamma^*)}{-c(1-\lambda)} > 0$ since $\nu(1) = 0$. It follows that all pairs (γ, B) that are better than $M(1, 0)$ are above and beyond $(\gamma^*, B^*) = (1, 1)$, i.e., are unfeasible. Therefore, only $(\beta_0, \beta_1) = (1, 0)$ is optimal for any $c > 0$.

There are also some parameter configurations for which the unique optimal strategy is strictly mixed. Let's return to the example in Fig. 2 introduced in the proof of Proposition 6. The cost of replacement is high enough ($c = 1.5$, to be compared with a maximum bribe of 1) that if $\kappa = 0.26$ the electorate chooses the incentive strategy. However, for a larger κ it would prefer not to replace the corrupt politician every time she is caught taking a bribe, because that would be too expensive. Instead, it would choose to mimic a smaller κ by way of the strategy described in the proof of Lemma 2. By increasing the probability of reelecting a politician that has been caught β_1 it renounces to give more stringent incentives to the corrupt politician, because incentives are just too costly. This is shown in Figure 5. We see in panel (a) that for $\kappa > 0.26$ the incentive strategy $(\beta_0, \beta_1) = (1, 0)$ becomes suboptimal. Panel (b) shows the same, this time by comparing the value functions for fixed electorate strategies. By increasing β_1 as κ increases beyond 0.26 the electorate is able to replicate both, the threshold and frequency of replacements, thereby maintaining the value W. In other words, a strictly mixed strategy is strictly preferred when the incentive strategy becomes suboptimal, in this example because of the high cost of replacement.

8 Long run outcomes

This section studies the long run outcomes generated by the Markov equilibrium under no commitment and under pure commitment.

Recall that corrupt politicians choose a cutoff γ, and the electorate chooses the probability of reelection of known corrupt politicians, conditional on the corrupt acts being exposed is β_0, while it is β_1 if not exposed. For a strategy γ of the corrupt politician the probability that corrupt acts are perpetrated and exposed if a corrupt politician is in office is $\kappa\,(1 - F(\gamma))$, so for the optimal strategy $\hat{\gamma}$, the probability is $\kappa\,(1 - F(\hat{\gamma}))$. By contrast to $\hat{\gamma}$ when $(\beta_0, \beta_1) = (1, 0)$ the optimal cutoff is denoted by γ^*. Recall also that the probability of a new politician being honest is η_0 and that the probability of the honest politician of remaining honest next period is λ. The



(a) $c = 1.5$.

(b) Payoffs as a function of κ.

Figure 5: Optimal mixed strategy with commitment

This numerical example is the same as Figure 2,'s namely a uniform distribution F over u and $(c, \delta, \eta_0, \lambda, \sigma) = (1.5, 0.95, 0.2, 0.8, 1)$. Under these assumptions, $c_1 = \frac{19}{48}$. Left panel: the choice of $\left(\hat{\gamma}, \hat{B}\right)$ with an optimum in a wall rather than a vertex of the feasible set, corresponding to an interior β_1. Right panel: the value functions for different strategies $P(\beta_0, \beta_1, \eta)$ for $\beta_0 = 1$ and $\eta = 0$ (i.e., the electorate facing a known corrupt politician) for a bunch of values of β_1. In particular, the value of $c = \frac{3}{2}$ is high enough that under the incentive policy $(1, 0)$ further increases in κ beyond a point reduce the electorate's welfare. However, after that point a strategy that uses a higher value of β_1 becomes optimal. Indeed, the value function W is a envelope of such P functions in that domain segment. W is thus non decreasing in κ.

Proposition below characterizes what the long run probability of corrupt acts is as a function of the underlying parameters. This probability is the product of those of corrupt politicians being in office, and corrupt politicians committing corrupt acts.

Proposition 8. *The equilibrium frequency of corrupt acts is*

Frequency	*Without commitment if*	*With pure commitment if*
$\frac{1-\lambda}{1-\lambda+\eta_0}$	$c < c_1$	$c < c_0$
$\frac{1-\lambda}{1-\lambda+\eta_0\kappa(1-F(\gamma^*))}\left(1-F\left(\gamma^*\right)\right)$	–	$c_0 < c < c_2$
1	$c_1 < c$	$c_2 < c$

where c_1 is as defined in Eq. (20) *and c_0 and c_2 in Eqs.* (21a) *and* (21b), *respectively.*

Proposition 8 tells us how often corruption acts will be perpetrated in equilibrium as a function of the replacement cost c and the availability of a commitment technology. Without commitment, in the worse case, if c is too high, there will be complete corruption. At the other extreme, even if c is null there will always be a strictly positive frequency of corrupt acts because even honest politicians will eventually become corrupt. How large is that frequency depends on both the probability of honest politicians remaining hones λ and the probability of new politicians being honest η_0. With commitment, nothing changes for too low or too high costs (namely, $c < c_0$ and $c > c_2$, respectively). However, for intermediate-high cost levels ($c \in (c_1, c_2)$) the frequency of corrupt acts is smaller than without commitment as the corrupt politicians are incentivized to pass up the lesser bribe opportunities. For intermediate-low cost levels ($c \in (c_0, c_1)$) the frequency comparison is a priori unclear, because under the incentive strategy that would be used in equilibrium the electorate retains corrupt politicians that without commitment would have fired.

The proof of Proposition 8 is provided after the following Lemma.

Lemma 3 (Long run probabilities)**.** *For a fixed strategy profile $(\gamma, \beta_0, \beta_1)$ - equilibrium or otherwise - the long run frequency of corrupt politicians in office is*

$$\pi(\gamma, \beta_0, \beta_1) \equiv \frac{1-\lambda}{1-\lambda+\eta_0(1-\beta_0+\kappa\left(\beta_0-\beta_1\right)\left(1-F\left(\gamma\right)\right))}. \tag{33}$$

while the long run frequency of corrupt acts is $\pi(\gamma, \beta_0, \beta_1)\left(1-F(\gamma)\right)$.

Proof. In the Appendix. □

Proof of Proposition 8. Recall that in equilibrium: (i) Without commitment, $\hat{\beta}_0 = \hat{\beta}_1 = 0$ if $c < c_1$ and $\hat{\beta}_0 = \hat{\beta}_1 = 1$ if $c > c_1$ and $\hat{\gamma} = 0$ in either case; (ii) With pure commitment, $\hat{\beta}_0 = \hat{\beta}_1 = 0$ and $\hat{\gamma} = 0$ if $c < c_0$; $\hat{\beta}_0 = 1$, $\hat{\beta}_1 = 0$ and $\hat{\gamma} = \gamma^*$ if $c_0 < c < c_2$; and $\hat{\beta}_0 = \hat{\beta}_1 = 1$ and $\hat{\gamma} = 0$ if $c > c_2$. Evaluating the frequency $\pi\left(\hat{\gamma}, \hat{\beta}_0, \hat{\beta}_1\right)$ from Lemma 3 times $(1 - F(\hat{\gamma}))$ yields Proposition 8. $\square$

While committing to the policy $(\beta_0, \beta_1) = (1, 0)$ yields an intermediate frequency of in-office corrupt politicians it might be that this strategy minimizes the frequency of corrupt *acts*. This, in view of the fact that corrupt politicians will find it optimal to pass up the smaller bribing opportunities. Indeed, under the incentive policy the long run frequency of corrupt acts is smaller than under the policy of removing corrupt politicians if $\eta_0 < \frac{1-\lambda}{1-\kappa}\frac{F(\gamma^*)}{1-F(\gamma^*)}$.

We conclude with an example where the long run probability of corrupt politicians in office and of corrupt acts are both not monotone in κ - against intuition -. One could expect that improvements in the monitoring technology would allow for a finer fight against corruption and indeed it does, except that getting rid of corrupt politicians too often may become excessively expensive. Consider again the case of the uniform distribution over u with the parameters in Figure 5b. It can be appreciated there that for $\kappa = 0$ and slightly above, the optimal policy is $(1, 1)$ because the cost of replacement is too high – above the threshold c_2. Under that policy, both long run probabilities are 1. However, as the monitoring technology κ improves, at some point the incentive strategy becomes optimal and both long run probabilities drop. Still, as κ continues increasing, eventually the incentive strategy requires getting rid of the corrupt politicians way too often. Under pure commitment, the optimal strategy is to switch back to $(1, 1)$, with both long run probabilities going back to 1, completing a discontinuous U-shape pattern. It should be stressed that this non-monotonicity may also present itself under mixed-commitment.

9 Fighting corruption: Discussion

Our main conclusion is that the ability to commit to a reelection policy is essential for elections to be of any help against corruption. So, what commitment technologies do exist?

We need to distinguish the two forms of commitment that emerge in our model. On the one hand, there is the commitment to throw out a politician that was caught taking bribes. This might be achievable by means of a (not uncommon) constitutional provision that makes office incompatible with a conviction. Under this interpretation, exposing corruption would mean proving it in a court of law. The parameter κ would measure the quality of the judicial system, in terms of how likely it is to produce convictions to guilty parties.[2] Such a constitutional proviso, however, cannot be expected to work in all circumstances. For instance, popular strongmen are known for their proclivity to change constitutions.

Another form of commitment is involve public rage. Human emotions have been interpreted as commitment devices elsewhere both, in the context of individuals as well as communities (e.g., Aubé (1998) and Berezin (2002)). In this case, κ would measure the quality of the process by which the truth about the politicians' behavior reaches the public as well as the public's willingness to act upon such information. This is a form of community enforcement that would come about by a combination of a free press and an interested or involved electorate.

Another point in favor of this interpretation is that the press is more inclined to cover immediate, short run situations or events, rather than making balanced, long run evaluations of careers or systems. This is exactly what a Markov strategy calls for: no matter the past, the electorate just needs to know whether in this period the corrupt politician executed the corrupt act or not.

The second form of commitment that is required in our model is the commitment to retain a known corrupt politician whose bribe taking, if any, has not being exposed. Again, if the electorate were to get rid of corrupt politicians regardless of their actions, they would take all bribes. Incentivizing corrupt politicians to choose carefully their opportunities requires both forms of commitment. Although this form of commitment seems to be trickier to achieve it seems that known corrupt politicians are generally re-elected in practice. For example, the American politician Huey Long, was widely believed and we think "known" to be corrupt, but charges were never proven (specific bribes not observed) and he was continually re-elected until his assassination.

The cost of firing the incumbent is not entirely independent of the commitment technology in place. Public rage, for instance, could be very destructive. A mature political system should seek less costly options. Indeed, modern democracies consider impeachment and elections as mechanisms for expelling an incumbent, the former generally considered as being more expensive than the latter. Indeed, Gins-

[2] Our model assumed that it is impossible that the innocent is found guilty, an error which surely also occurs in real life.

burg, Huq, and Landau (2020) point out that impeachment in practice is not used to get rid of corrupt politicians but rather to solve political crises. Our analysis suggests that developing cheaper ways of replacing politicians can contribute greatly to the welfare of the electorate.

The parameter η_0, namely, the probability that a new politician is honest, also plays a crucial role in the equilibrium we study. The larger it is, the higher the costs needed for the incentive strategy to be optimal. Indeed, in a country with honest politicians the best the electorate can do is getting rid of the corrupt ones as soon as they are spotted. Our incentive strategy only makes sense when the situation is not as good, and honest politicians are not as likely to come by.

10 Conclusions

We studied a model of a rational electorate who faces the risk of being wronged by corrupt politicians. The defense tools we considered were a combination of elections with the imperfect monitoring of the in-office politician's actions. We concluded that without the ability to commit, elections provide no effective defense against corruption. On the other hand, the ability to commit may not prevent corruption altogether, but can alleviate the problem somewhat, by means of inducing corrupt politicians to pass by bribing opportunities in the hope of being in office when a better one presents itself.

The ability to commit works hand in hand with a monitoring technology, which we assumed was imperfect. To our surprise, we found that even though monitoring is essential for making electoral accountability possible, improvements in monitoring not necessarily make the electorate better off. It all hinges on the way corrupt politicians react to an improved surveillance system, as well as the increased cost of replacing corrupt politicians.

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A Proofs

Proof of Lemma 1. From Eq. (3), the condition for $a_t = 1$ to be optimal is that

$$u_t \geq \kappa \frac{\delta}{1-\delta} \left(\beta_0 V - \beta_1 V\right). \tag{34}$$

Let us define $\hat{\gamma}$ as in Eq. (6). This expression is independent of u_t and as such, it is a cutoff. $a_t = 1$ is the unique optimum if $u_t > \hat{\gamma}$. □

Proof of Proposition 2. Any prior η_t transforms after type revelation into either $\underline{\eta}_t = 0$ or $\underline{\eta}_t = 1$. In turn, at the electorate's time of choice either the bribe taking act was revealed ($k_t = 1$) or not ($k_t = 0$). The value function $w(h_t, k_t)$ in Eq. (16) thus becomes:

$w(h_t, k_t)$	$(\beta = 0),\ (\beta = 1)$
$w(1,0) = \max\{$	$-c + \delta W(\eta_0),\ \delta W(\lambda)\}$
$w(0,0) = \max\{$	$-(1-\delta)(\sigma\nu_0 + c) + \delta W(\eta_0),\ -(1-\delta)\sigma\nu_0 + \delta W(0)\}$
$w(0,1) = \max\{$	$-(1-\delta)(\sigma\nu_1 + c) + \delta W(\eta_0),\ -(1-\delta)\sigma\nu_1 + \delta W(0)\}$

where $\nu_0 \equiv E[ua|k=0]$, $\nu_1 \equiv E[ua|k=1]$, $\nu \equiv E[ua]$ and

$$W(\eta) = \eta w(1,0) + (1-\eta)\left(\left(1-(1-F(\gamma))\kappa\right) w(0,0) + (1-F(\gamma))\kappa w(0,1)\right). \tag{35}$$

Observe that:

The state $(h_t, k_t) = (1,1)$ is a zero probability event - it is impossible for Nature to expose a bribe that was not taken, since honest politicians do not take bribes. Hence, $w(1,1)$ is not part of $W(\eta)$.

The functions $w(1,0)$, $w(0,0)$, and $w(0,1)$ are all independent of η; hence, the value function $W(\eta)$ in (35) is linear. Moreover, $W'(\eta) = W(1) - W(0)$. This is to say, the value function is increasing if and only if the value of facing a honest politician is greater than that of facing a corrupt one - a fact that we will prove shortly.

A sufficient condition for keeping the known honest politician is $\lambda > \eta_0$, which what we assumed. Indeed, at states $(h_t, k_t) = (1,0)$, $\beta = 1$ is optimal if $-c + \delta W(\eta_0) \leq \delta W(\lambda)$. If $W(\eta)$ is increasing, $\lambda > \eta_0 \implies \delta W(\lambda) > \delta W(\eta_0) > \delta W(\eta_0) - c$. Hence, the honest politician is reelected.

The choice of reelecting or replacing the corrupt politician must be the same regardless of whether the politician was caught or not taking a bribe, that is: $\beta_0 = \beta_1$. This is because the conditions for firing the corrupt politician to be optimal are the same in both states in spite of the fact that the values are different:

$$\begin{gathered} -(1-\delta)(\sigma\nu_0 + c) + \delta W(\eta_0) \geq -(1-\delta)\sigma\nu_0 + \delta W(0) \iff \\ -(1-\delta)(\sigma\nu_1 + c) + \delta W(\eta_0) \geq -(1-\delta)\sigma\nu_1 + \delta W(0). \end{gathered}$$

It follows then, that if $\beta_0 = \beta_1 = 1$ (the always replace strategy) were optimal when facing a corrupt politician, the value function would evaluate to

$$W(\eta) = \frac{\sigma\nu(1-\delta)}{1-\delta\lambda}\left(-\frac{1-\delta\lambda}{1-\delta} + \eta\right). \tag{36}$$

If $\beta_0 = \beta_1 = 0$ (the always keep strategy) were optimal when facing a corrupt politician, then

$$W(\eta) = \frac{(c+\sigma\nu)(1-\delta)}{1-\delta\lambda+\delta\eta_0}\left(-\frac{1-\delta\lambda}{1-\delta} + \eta\right). \tag{37}$$

Lastly, if $\beta_0 = 1,\ \beta_1 = 0$ (the incentive strategy) were optimal when facing a corrupt politician,

$$W(\eta) = \frac{(c\kappa(1-F(\gamma)) + \sigma\nu)(1-\delta)}{1-\delta\lambda+\delta\eta_0\kappa(1-F(\gamma))}\left(-\frac{1-\delta\lambda}{1-\delta} + \eta\right). \tag{38}$$

The three candidate value functions (36), (37), and (38) are strictly increasing in η.

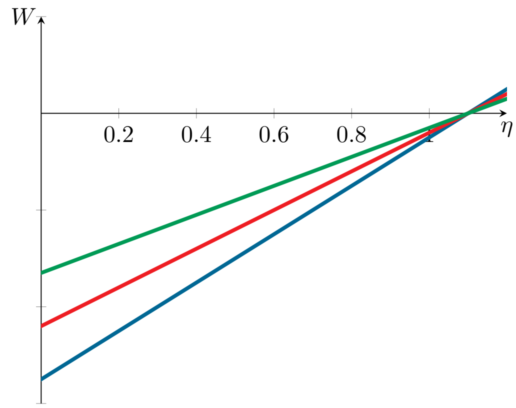

Figure 6: Electorate's value function

Moreover, they evaluate to zero at the same point ($W(\frac{1-\delta\lambda}{1-\delta}) = 0$, with $\frac{1-\delta\lambda}{1-\delta} > 1$) so that each function is greater than the other for all $\eta \in [0,1]$ if and only if its slope is smaller, as Figure 6 shows.

Hence, comparing slopes pairwise we deduce that:

$$(\beta_0, \beta_1) = (1,1) \succcurlyeq (1,0) \iff c \geq \frac{\delta\eta_0}{1-\delta\lambda}\sigma\nu\left(\gamma\right) \equiv \hat{c}\left(\gamma\right);$$

$$(1,1) \succcurlyeq (0,0) \iff c \geq \frac{\delta\eta_0}{1-\delta\lambda}\sigma\nu\left(\gamma\right); \text{ and}$$

$$(1,0) \succcurlyeq (0,0) \iff c \geq \frac{\delta\eta_0}{1-\delta\lambda}\sigma\nu\left(\gamma\right).$$

In other words: either the cost of replacing is too high ($c > \hat{c}$), and all politicians are always kept (policy $(1,1)$ is chosen), or it is low enough so that corrupt politicians are always replaced (policy $(0,0)$ is chosen). In particular, the incentive strategy is not optimal except when the cost is $\hat{c}$ and is indifferent among all strategies. $\square$

Proof of Proposition 4. The interval (c_0, c_2) is nonempty and it contains c_1^*. Indeed, since $\nu\left(\gamma\right)$ is decreasing, $\nu\left(0\right) > \nu\left(\gamma^*\right)$ and therefore

$$\begin{aligned} c_0 &= \frac{1}{1-\delta\lambda}\frac{\sigma}{1-\kappa\left(1-F\left(\gamma^*\right)\right)}\Big(\nu\left(\gamma^*\right)\left(1-\lambda\delta+\delta\eta_0\right)-\nu\left(0\right)\left(1-\delta\lambda+\delta\eta_0\kappa\left(1-F\left(\gamma^*\right)\right)\right)\Big) \\ &< \frac{\sigma\delta\eta_0}{1-\delta\lambda}\nu\left(\gamma^*\right) = c_1^*, \end{aligned} \tag{39}$$

while

$$c_2 = c_1 + \frac{\sigma}{\kappa\left(1 - F\left(\gamma^*\right)\right)}\left(\nu\left(0\right) - \nu\left(\gamma^*\right)\right) > c_1 > c_1^*. \tag{40}$$

The fact that c_0 and c_2 are both proportional to σ is evident from Eqs. (21a) and (21b).

On the other hand,

$$\frac{\partial c_0}{\partial \eta_0} = \frac{1}{1 - \delta\lambda} \frac{\delta\sigma}{1 - \kappa\left(1 - F\left(\gamma^*\right)\right)}\big(\nu\left(\gamma^*\right) - \nu\left(0\right)\kappa\left(1 - F\left(\gamma^*\right)\right)\big).$$

We can write

$$\nu\left(\gamma\right) \equiv \int_\gamma^1 uf\left(u\right)du = \int_0^1 uf\left(u\right)du - \int_0^\gamma uf\left(u\right)du$$

Recognizing that the first integral is $\nu\left(0\right)$ and integrating by parts the second integral, we get

$$\int_0^\gamma uf\left(u\right)du = \gamma F\left(\gamma\right) - \int_0^\gamma F\left(u\right)du < \gamma F\left(\gamma\right)$$

where the inequality follows from the fact that the second integral in this latter expression is positive. Hence,

$$\nu\left(\gamma\right) + \gamma F\left(\gamma\right) > \nu\left(0\right).$$

Evaluating $\gamma = \gamma^*$, using Eq. (15), and simplifying we see that

$$\nu\left(\gamma^*\right) > \nu\left(0\right)\frac{1 - \delta + \delta\kappa\left(1 - F\left(\gamma^*\right)\right)}{1 + \delta + \delta\kappa} > \nu\left(0\right)\left(1 - F\left(\gamma^*\right)\right) > \nu\left(0\right)\kappa\left(1 - F\left(\gamma^*\right)\right).$$

It follows that $\frac{\partial c_0}{\partial \eta_0} > 0$. Also, $\frac{\partial c_0}{\partial \lambda} > 0$ follows the same fact since

$$\frac{\partial c_0}{\partial \lambda} = \frac{\delta^2}{\left(1 - \delta\lambda\right)^2} \frac{\sigma\eta_0}{1 - \kappa\left(1 - F\left(\gamma^*\right)\right)}\big(\nu\left(\gamma^*\right) - \nu\left(0\right)\kappa\left(1 - F\left(\gamma^*\right)\right)\big).$$

On the other hand, $\frac{\partial c_2}{\partial \eta_0} > 0$ is immediate from Eq. (21b).

Finally, since $\lim_{\delta \to 0} \gamma^* = 0$, we have

$$\lim_{\delta \to 0} c_2 = \frac{\sigma}{\kappa\left(1 - F\left(\gamma^*\right)\right)}\left(\nu\left(0\right) - \nu\left(\gamma^*\right)\right) = 0.$$

$\square$

Proof of Proposition 5. The electorate's policy is $\beta_0 = 1$ and $\beta_1 = 0$, since $c \in (c_0, c_2)$. Specializing Eq. (11) to it, we get

$$\frac{\mathrm{d}\hat{\gamma}}{\mathrm{d}\delta} = \frac{\partial g}{\partial \delta} = \frac{\kappa}{1-\delta+\delta\kappa\left(1-F\left(\gamma^*\right)\right)}\nu\left(\gamma^*\right) > 0. \tag{41}$$

As for the second part, observe that taking limits at both sides of (15), we get

$$\lim_{\delta\to 1}\gamma^* = \lim_{\delta\to 1} g(\gamma^*) = \frac{\nu\left(\gamma^*\right)}{\left(1-F\left(\gamma^*\right)\right)} = E\left[u|u \geq \gamma^*\right].$$

The resulting equation requires $\gamma^* = 1$ because f has full support. Finally, $\lim_{\delta\to 0} g(\gamma^*) = 0$. $\square$

Proof of Lemma 3. The Markov chain on moral types induced by the strategies $(\gamma, \beta_0, \beta_1)$ is characterized by the following transition matrix

$$\begin{pmatrix} 1-\eta_0+\eta_0\left(\beta_0\left(1-\kappa\left(1-F(\gamma)\right)\right)+\beta_1\kappa\left(1-F(\gamma)\right)\right) & 1-\lambda \\ \eta_0-\eta_0\left(\beta_0\left(1-\kappa\left(1-F(\gamma)\right)\right)+\beta_1\kappa\left(1-F(\gamma)\right)\right) & \lambda \end{pmatrix}, \tag{42}$$

where the columns are the states at t (corrupt, honest), and the rows at $t+1$ (corrupt, honest).

Indeed, if the politician at t is honest ($h_t = 1$), she gets reelected with probability 1 and Nature keeps her honest with probability λ. If she is corrupt, she takes the bribe and is caught with probability $\kappa\left(1-F(\gamma)\right)$, and she is reelected with probability β_1 or replaced by another corrupt politician with probability $(1-\beta_1)(1-\eta_0)$. With probability $\left(1-\kappa\left(1-F(\gamma)\right)\right)$ she is not caught (either because she didn't take the bribe or because she did but was not caught), and is reelected with probability β_0 or replaced by another corrupt politician with probability $(1-\beta_0)(1-\eta_0)$. Hence,

$$\begin{aligned}\Pr(h_{t+1}=0|h_t=0) = \kappa\left(1-F(\gamma)\right)\left(\beta_1+(1-\beta_1)(1-\eta_0)\right) \\ +\left(1-\kappa\left(1-F(\gamma)\right)\right)\left(\beta_0+(1-\beta_0)(1-\eta_0)\right).\end{aligned} \tag{43}$$

Simplifying, we get

$$\Pr(h_{t+1}=0|h_t=0) = 1-\eta_0+\eta_0\left(\beta_0\left(1-\kappa\left(1-F(\gamma)\right)\right)+\beta_1\kappa\left(1-F(\gamma)\right)\right). \tag{44}$$

On the other hand, it is a well known property of two-state Markov chains that the long run frequency of each state is given by the ratio of the probability of a switch from the opposite state over the sum of the two switching probabilities. Hence, the long run (or steady state) probability that the politician in office is honest is given by the probability that the honest becomes corrupt, over the probability that either type is replaced by a politician of a different type the next period. From the constructed transition matrix we select the anti-diagonal terms. After simplifying the denominator, we get:

$$\pi(\gamma, \beta_0, \beta_1) \equiv \frac{1-\lambda}{1-\lambda+\eta_0(1-B)},$$

as asserted.

Finally, the probability of a corrupt act is the product of the probability of a corrupt politician being in office π and the probability that a corrupt politician commits a corrupt act $(1 - F(\gamma))$. □

B Non strict equilibria without commitment

In the no commitment case, the electorate is indifferent among all behavior strategies (β_0, β_1) if the cost of replacing the politician is exactly as in Eq. (19), namely

$$\hat{c}\left(\gamma\right) = \frac{\delta\eta_0}{1-\delta\lambda}\sigma\nu\left(\gamma\right).$$

On the other hand, $\hat{\gamma}$ is determined as a best response to the strategy (β_0, β_1) by the implicit Eq. (10), namely

$$\hat{\gamma} = \frac{\kappa\delta\left(\beta_0-\beta_1\right)}{1-\delta\left(\beta_0-\left(1-F\left(\hat{\gamma}\right)\right)\kappa\left(\beta_0-\beta_1\right)\right)}\nu\left(\hat{\gamma}\right).$$

Then, each (β_0, β_1) determines a unique $\hat{\gamma}\left(\beta_0, \beta_1\right)$. This function has associated level curves $\gamma = \hat{\gamma}\left(\beta_0, \beta_1\right)$ ranging from 0 to γ^* (its image set). In turn, as we just said, each $\hat{\gamma}$ makes the electorate indifferent among all strategies only at the unique cost $\hat{c}\left(\gamma\right) = \frac{\delta\eta_0}{1-\delta\lambda}\sigma\nu\left(\hat{\gamma}\left(\beta_0, \beta_1\right)\right)$. Figure 7 depicts the intersection, or equilibrium correspondence. The vertical axis shows costs c and the horizontal axes the electorate strategies (β_0, β_1). The correspondence is the set of all electorate strategies that are best responses to the induced $\hat{c}\left(\hat{\gamma}\left(\beta_0, \beta_1\right)\right)$ all cost levels at which the electorate is indifferent among all strategies a subset of strategies, intersected with the strategies that generate the said indifference. When more than one strategy

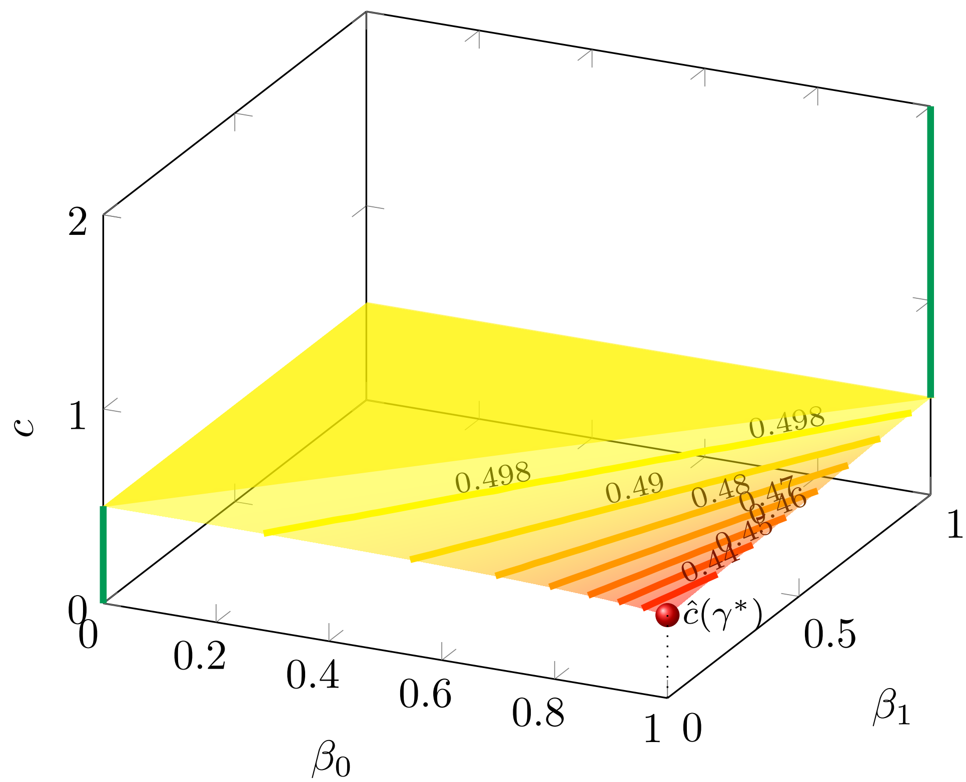

Figure 7: Equilibrium correspondence, no commitment game
This example assumes a uniform distribution over u and parameters $(\kappa, \delta, \eta_0, \sigma, \lambda) = \left(\frac{1}{2}, \frac{8}{10}, \frac{1}{2}, 1, \frac{3}{4}\right)$, from which $c_1 = 0.5$. For each cost of replacement $c \in (0, 2)$ the plot shows the equilibrium pairs (β_0, β_1). Strict equilibria are shown in green solid lines.: if $c < \frac{1}{2}$ the electorate strictly prefers the behavior strategy $(0, 0)$ and the corrupt politician strictly prefers $\hat{\gamma} = 0$; if $c > \frac{1}{2}$ the electorate strictly prefers the behavior strategy $(1, 1)$ and the corrupt politician strictly prefers $\hat{\gamma} = 0$. If $c_1^* = 0.427051 < c < 0.5 = c_1$ there are equilibria in which the electorate is indifferent among all strategies, including some that would induce positive levels of γ on the politician. In particular, if $c = \frac{1}{2}$ the electorate is indifferent among all strategies but among them, all such that $\beta_1 \geq \beta_0$ induce the corrupt politician to choose $\hat{\gamma} = 0$ (the yellow triangle); by the same token, the strategy $(\beta_0, \beta_1) = (1, 0)$ induces $\gamma = \gamma^*$ at which the electorate is indifferent among all strategies if $c = c_1^* = 0.427051$ (the red dot). Observe that all equilibria in combinations of red and yellow occur with the electorate under complete indifference and therefore are not strict.