A General Equilibrium Model of Corruption

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Abstract

This paper develops a General Equilibrium model to quantify the economic cost of corruption by incorporating it as a distinct activity within a closed economy. Unlike traditional approaches that treat corruption as an exogenous friction, our model endogenizes the choice between productive labor and rent-seeking activities. Specifically, we introduce an auxiliary technology that captures the role of corruption in diverting labor away from the formal sector and distorting resource allocation, thereby reducing overall economic output. Our findings reveal that corruption can reduce the steady-state output by over 4%, highlighting its significant impact on economic welfare. The model also suggests that policies such as reduced labor taxes or improved oversight could mitigate these losses by discouraging the shift of labor into unproductive corruption activities. This framework offers a new perspective on the costs of corruption and serves as a tool for policymakers to evaluate interventions that could enhance productivity by curbing rent-seeking incentives.

Keywords: Corruption, General Equilibrium, Rent-Seeking.

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1 Introduction

Corruption is a frequent practice in most countries in the world. Although many economists are skeptical about the measurable estimation of the causal relationship between corruption and output, most of them agree that the effects imply a non-negligible amount. An accepted 5% of global GDP, is lost to corruption annually around the world, although this number has never been estimated with precision.

The classical approach to study the aggregate effects of corruption is to analyze how it affects growth. As in Mauro (1995), corruption affects the output via private investment rate as the main channel. In his seminal paper, he presents the estimation of how much corruption lowers output. This is due to the malfunction of government institutions. A one-standard-deviation increase (improvement) in the corruption index[1] is associated with an increase in the investment rate by 2.9 percent of the GDP. In the Latin-American and the Caribbean context, we have the estimation developed in Carvalho, Tigre & de Paulo (2022). They show that a one standard deviation increase in corruption, as measured by the reversed Transparency International's corruption perception index, is associated with a decrease of 12.2% in gross domestic product per capita and a decrease of 3.05% in economic growth.

There is another branch of the literature that associates corruption and lobby with the bureaucratic capacity of the government, as if corruption would have an effect on the output through another means. The influence of bureaucratic authority structures on fostering economic growth has been a topic of sociological interest since Max Weber's foundational work nearly a century ago. While some suggest that bureaucracy affects negatively efficiency as it is a heavy regulatory environment foreign firms must face when seeking approvals and permits for investment as in Mauro (1995), others like in Acemoglu & Verdier (2000), believe bureaucracy could increase efficiency as it is an important tool to cope with market failures:

[1]The corruption index is taken from the Business International (BI), now incorporated into The Economist Intelligence Unit, a private firm that sells these indices to banks, multinational companies and other international investors.

government intervention requires "bureaucrats" to gather information and implement policies more efficiently.

In this paper, unlike the literature presented previously, we present a novel way of modeling the effect corruption has in a general equilibrium model. We pretend to understand in which way does corruption affect the steady state of the different variables affecting the economy, not only its growth. Corruption will be an activity in the economy which will not contribute to the final output. Our approach will be similar as in Bhagwati (1982). Corruption will be a directly unproductive rent-seeking activity. The micro problem that will affect the output will be related to the fact that each individual will have the choice of employing itself in the main formal activity or to choose to participate in corruption activities. The choice of participating in the corruption activity will be the one affecting the final output, as in Krueger (1974). It is in the capacity of each country to deal with corruption that the corruption participation will be big or small related to the main activity firm's labor. Corruption will have a technology to produce these spurious activities and will not affect the final output. This technology will have a shock component that will affect all the variables. We will model this shock as the coming to power of a corrupt government and how this will affect the country's productive credibility in the short and long term. As in Angeletos & Kollintzas (2000), we will observe that higher corruption will imply higher benefits for those employed in the rent-seeking activity. This will be presented as the main obstacle to the elimination of corruption.

The main activity firm will be modeled in a classical way. It will be a constant return-to-scale firm that will require capital and labor to produce the final output and will have a technology shock that affects positively the final output. As usual, this shock will represent a Total Factor Productivity (TFP) innovation and will have an impact in the level of corruption and in the other aggregate variables in the economy.

Our economy will have a government that will collect labor taxes on the main activity and

return them to the agent in transfers. The government won't be in charge of producing corruption as in many models. Corruption generated by the government will be seen in the form of this labor tax. The trade off the agent will have will be in working in the main sector and having the after-tax salary or participating in corruption activities whose salary does not pay labor tax as it cannot be observed by the government. A trivial result this paper will bring will be that to seize the effects of corruption, the government will have to lower the labor tax and correct that incentive scheme. We will not focus on the amount of public spending as a percentage of GDP, as is often the case in the literature, but rather on the portion of government revenue allocated to corrupt activities.

2 The Model

We consider an economy populated by a continuum of i (types of) infinitely-lived agents where $i \in [0, I]$ represents a typical consumer. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ where $T = \infty$ (infinite horizon). There is only one consumption good which can be either consumed or invested in the capital market.

Each agent is the owner of a representative firm and is endowed with one unit of time every period. They can use this unit for leisure, to work in the main activity or to perform corruption activities. Thus, h_i represents hours of leisure, L_i^w represents labor supplied by each agent at each unit of time t in its main activity and we will call L_i^c the unproductive hours spent in corruption activities for agent i at time t.

So, normalizing time to 1, we have that:

$$h_i^i + (L_i^i)^w + (L_i^i)^c = 1$$

The representative consumer's preferences on $\mathcal{C}$ are given by time-separable discounted utility.

That is, if $(C_i, h_i) \in \mathcal{C}$ then:

$$U\left(C_t^i, h_t^i\right) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^i, h_t^i\right)$$

where $u : R_+ \times [0,1] \to R_+$ is strictly increasing with respect to consumption C and leisure h, strictly concave and differentiable such that $(C, h) > 0$. Let E_0 represent the household's expectations at the beginning of period 0. In period t, C_t^i denotes the household's private consumption. The parameter β, where $0 < \beta < 1$, is the household's discount factor.

At the outset, the household has an initial stock of physical capital, $K_0 > 0$, and must decide how much new investment to undertake. The evolution of physical capital follows the equation:

$$K_{t+1}^i = X_t^i + (1 - \delta) K_t^i$$

where $0 < \delta < 1$ is the capital depreciation rate. The household earns capital income based on the real interest rate, r_t, resulting in capital income of $r_t K_t$ in period t.

In addition to capital income, the household earns labor income. Hours worked for the representative firm are compensated at the wage rate w_t, while the corruption activity also provides an income. We will go into more detail about that later. The government can only tax the labor income from the main sector. Finally, since each household owns a percentage of the firm, it is entitled to the firm's profits, let Π_t^i be the percentage of benefits of the firm owned by agent i. Since the firm will have constant returns to scale, these profits will be equal to 0 in equilibrium.

2.1 Corruption Process

The corruption process is characterized by a technology owned by each agent that is increasing and concave at each unit of time t. It is unproductive in terms of the consumption good so in the aggregate goods market equation, it won't have any impact. We will call it $\Psi_t(L_t^c)$.

This function won't use capital for production.

2.2 Budget Constraint

Each agent will face a trade-off between time spent in leisure, corruption and work on their main activity. They can rent units of capital at a risk free interest rate to the capital market a proportion to be saved each period. They also face a labor tax on working in the main activity.

Its budget constraint at time t will be:

$$C_t^i + X_t^i = r_t K_t^i + (1-\tau)(L_t^i)^w w_t + \Psi_t^i((L_t^i)^c) + \underbrace{\Pi_t^i}_{=0}$$

$$X_t^i = K_{t+1}^i - (1-\delta)K_t^i$$

$$h_t^i + (L_t^i)^w + (L_t^i)^c = 1$$

2.3 Agent's Problem

Each household will maximize its utility subject to its budget constraint each period. It will face the following maximization problem:

$$\max_{\left(C_t^i, h_t^i, (L_t^i)^w, (L_t^i)^c, K_{t+1}^i\right)} E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^i, h_t^i\right)$$

subject to:

$$C_t^i + X_t^i = r_t K_t^i + (1-\tau)(L_t^i)^w w_t + \Psi_t^i((L_t^i)^c)$$

$$X_t^i = K_{t+1}^i - (1-\delta)K_t^i$$

$$h_t^i + (L_t^i)^w + (L_t^i)^c = 1$$

The first order conditions to this problem if we call λ_t^i the Lagrange multiplier for the budget constraint of each agent i will be:

$$\begin{aligned}
\{C_t^i\} \quad & \lambda_t^i = u_c(C_t^i, 1 - (L_t^i)^w - (L_t^i)^c) \\
\{(L_t^i)^w\} \quad & u_h(C_t^i, 1 - (L_t^i)^w - (L_t^i)^c) = -\lambda_t^i(1-\tau)w_t \\
\{(L_t^i)^c\} \quad & u_h(C_t^i, 1 - (L_t^i)^w - (L_t^i)^c) = -\lambda_t^i \Psi_t^{i\prime}((L_t^i)^c) \\
\{K_{t+1}^i\} \quad & \lambda_t^i = \beta \lambda_{t+1}^i (1 - \delta + r_{t+1}) \\
\{\lambda_t^i\} \quad & C_t^i + X_t^i = r_t K_t^i + (1-\tau)(L_t^i)^w w_t + (L_t^i)^c w_t \\
\{TC_K^i\} \quad & \lim_{t\to\infty} \beta^t \lambda_t^i K_{t+1}^i = 0
\end{aligned}$$

We will assume a particular form of the corruption production function $\Psi_t(L_t^c) = \Upsilon_t \, (L_t^c)^{1-\varphi}$, with Υ_t being the corruption shock. We could interpret this shock as the unexpected seizure of power of a political party with a historical corrupt agenda. While in most of the literature the productivity of corruption Υ_t is assumed to be constant over time and known to the public, we assume in this paper that it is stochastic from the perspective of the public. Our model provides no explanation for the fundamental determinants of Υ_t. We assume that there are periods in time when corruption is low, $\Upsilon_t = \Upsilon^L$, and periods when it is high, $\Upsilon_t = \Upsilon^H$.

More specifically, the log productivity of corruption evolves according to a two-state Markov-switching process:

$$\ln \Upsilon_t = \begin{cases} \ln \Upsilon^L & \text{if } s_t = 1 \\ \ln \Upsilon^H & \text{if } s_t = 2 \end{cases}$$

where we will assume that $\ln \Upsilon^H \geq \ln \Upsilon^L$, and with transition matrix:

$$\mathcal{P} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix}$$

where:

- $\phi_1 = \Pr(s_{t+1} = L \mid s_t = L)$ is the probability of remaining in the low-corruption regime.
- $\phi_2 = \Pr(s_{t+1} = H \mid s_t = H)$ is the probability of remaining in the high-corruption regime.

A higher value of ϕ_1 implies greater persistence of the low-corruption state, whereas a higher ϕ_2 implies greater persistence of the high-corruption state. These transition parameters are key to determining the ergodic distribution of regimes and therefore affect the long-run average productivity in the economy.

The state s_t is unobserved by the public, and hence agents have to learn from past observables whether the economy is currently in a high-corruption or low-corruption regime. In particular, they form beliefs about the prevailing corruption regime by observing variables such as wages, tax revenues, or labor allocations. These beliefs affect their expectations of the productivity of corruption Υ_t and thus influence their decisions regarding labor supply and capital expenditure.

Now, we assume a particular form for the preferences and the production function and find the equilibrium. The parameter η is a constant affecting the disutility of working. We will use a standard Cobb-Douglas separable utility form. In this setup, we know that we can represent the problem of all the households with a representative agent (see proof in Appendix 7.1).

$$U\left(C_t, h_t\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \eta \ln h_t\right]$$

So, the previous FOC now reduce to:

$$\{C_t\} \quad \lambda_t = \frac{1}{C_t} \tag{1}$$

$$\{L_t^w\} \quad \frac{\eta}{1 - L_t^w - L_t^c} = \lambda_t (1 - \tau) w_t \tag{2}$$

$$\{L_t^c\} \quad \frac{\eta}{1 - L_t^w - L_t^c} = \lambda_t \Upsilon_t (1 - \varphi) \left(L_t^c\right)^{-\varphi} \tag{3}$$

$$\{K_{t+1}\} \quad \lambda_t = E_t \left(\beta \lambda_{t+1} (1 - \delta + r_{t+1})\right) \tag{4}$$

$$\{\lambda_t\} \quad C_t + X_t = r_t K_t + (1 - \tau) L_t^w w_t + \Upsilon_t \left(L_t^c\right)^{1-\varphi} \tag{5}$$

$$\{TC_K\} \quad \lim_{t \to \infty} \beta^t \lambda_t K_{t+1} = 0 \tag{6}$$

Taking the FOC with respect to $\{L^w\}$ and equating it with $\{L^c\}$ we get the standard non-arbitrage equality where the salary with taxes from the main activity must be the same as the corruption marginal revenue the agent has from that activity:

$$(1 - \tau) w_t = \Upsilon_t^i (1 - \varphi) \left((L_t^i)^c\right)^{-\varphi} \tag{7}$$

This condition helps us to rule out corner solutions in each sector.

Now, taking the FOC $\{C_t\}$ and $\{K_{t+1}\}$ we get the Euler condition for our model:

$$E_t \left[\beta \left(\frac{C_t}{C_{t+1}}\right) (r_{t+1} + 1 - \delta)\right] = 1 \tag{8}$$

$$\tag{9}$$

2.4 Representative Firm's Problem

As we mentioned before, there is a representative firm in the economy that produces the final product. The firm's production follows a Cobb-Douglas technology and uses both physical

capital, K_t, and labor hours of the agent's main activity, L_t^w, to maximize its profit at each point in time. The production function thus is economy is $Y_t = A_t K_t^{\alpha} \left(L_t^w\right)^{1-\alpha}$, where A_t represents the level of technology at time t. Technology evolves according to

$$\ln\left(A_{t+1}\right) = (1-\rho)\log(A) + \rho \ln\left(A_t\right) + \varepsilon_{t+1} \tag{10}$$

where $|\rho| < 1$ and $\varepsilon_t \sim N\left(0, \sigma_{\varepsilon}^2\right)$.

The price of this output is set to one for simplicity. The firm's profit function is:

$$\max_{L_t^w, K_t} \Pi_t = A_t K_t^{\alpha} \left(L_t^w\right)^{1-\alpha} - r_t K_t - w_t L_t^w \tag{11}$$

Since the firm rents capital from households, its problem is to maximize profit in each period independently.

$$\{K_t\} : \alpha \frac{Y_t}{K_t} = r_t \tag{12}$$

$$\{L_t^w\} : (1-\alpha) \frac{Y_t}{L_t^w} = w_t \tag{13}$$

In equilibrium, the firm earns zero profits, and each input is paid according to its marginal product, then, $\Pi_t = 0$ for all t.

2.5 The Government

We assume the Government levies labor tax to the Representative Agent and returns it to the agent in the form of the corruption function.

$$\Upsilon_t \left(L_t^c\right)^{1-\varphi} = \tau w_t L_t^w \tag{14}$$

3 Equilibrium

The equilibrium conditions for this problem are:

Factor markets

$$K_t^d = K_t^s = K_t$$

$$(L_t^w)^d = (L_t^w)^s = L_t^w$$

Market for goods

$$Y_t = C_t + X_t$$

$$Y_t = A_t K_t^{\alpha} \left(L_t^w\right)^{1-\alpha}$$

$$\Psi_t = \Upsilon_t \left(L_t^c\right)^{1-\varphi}$$

$$X_t = K_{t+1} - (1-\delta)K_t$$

Profit maximization of the representative firm.

$$r_t = \alpha \frac{Y_t}{K_t}$$

$$(1-\alpha)\frac{Y_t}{L_t^w} = w_t$$

Utility maximization

$$\frac{\eta}{1 - L_t^w - L_t^c} = \frac{1}{C_t}(1-\tau)w_t$$

$$\frac{\eta}{1 - L_t^w - L_t^c} = \frac{1}{C_t}\Upsilon_t(1-\varphi)\left(L_t^c\right)^{-\varphi}$$

$$E_t\left[\beta \frac{C_t}{C_{t+1}}\left(r_{t+1} + 1 - \delta\right)\right] = 1$$

Government Budget Constraint:

$$\Upsilon_t \left(L_t^c\right)^{1-\varphi} = \tau w_t L_t^w$$

Equilibrium in the market for goods together with the zero-profit condition and the government's budget constraint ensure that the problem will have a solution.

3.1 System

$$
\begin{aligned}
&Y_t = C_t + X_t \\
&Y_t = A_t K_t^{\alpha} \left(L_t^w\right)^{1-\alpha} \\
&\Psi_t = \Upsilon_t \left(L_t^c\right)^{1-\varphi} \\
&X_t = K_{t+1} - (1-\delta) K_t \\
&r_t = \alpha \frac{Y_t}{K_t} \\
&(1-\alpha) \frac{Y_t}{L_t^w} = w_t \\
&\frac{\eta}{1 - L_t^w - L_t^c} = \frac{1}{C_t} (1-\tau) w_t \\
&\frac{\eta}{1 - L_t^w - L_t^c} = \frac{1}{C_t} (1-\varphi) \frac{\Psi_t}{L_t^c} \\
&E_t \left[\beta \frac{C_t}{C_{t+1}} \left(r_{t+1} + 1 - \delta\right) \right] = 1 \\
&\Upsilon_t \left(L_t^c\right)^{\varphi} = \tau w_t L_t^w \\
&\ln\left(A_{t+1}\right) = (1-\rho) \log(A) + \rho \ln\left(A_t\right) + \varepsilon_{t+1} \\
&\Upsilon_t \in \{\Upsilon^L, \Upsilon^H\} \text{ with transition } \mathcal{P} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix}
\end{aligned}
$$

We have 11 equations per period to solve for the evolution of 11 variables: 9 endogenous variables $(K_t, Y_t, \Psi_t, C_t, X_t, L_t^w, L_t^c, r_t, w_t)$ and 2 exogenous variable (A_t, Υ_t). Of the 9 endogenous variables, one is a state variable (K_t) and nine are jump variables $(Y_t, \Psi_t, C_t, X_t, L_t^w, L_t^c, r_t, w_t)$. The model also includes two exogenous processes: total factor productivity of the main activity A_t, which follows an AR(1), and corruption productivity Υ_t, which follows a two-state Markov process. While A_t is driven by a stochastic innovation, the values and transition probabilities of Υ_t are calibrated parameters, reflecting the productivity of rent-seeking activities under low and high corruption regimes. For simplicity of notation, we do not explicitly write each variable as a function of the history of states (e.g., $C(s^t)$), and instead use C_t, with the

understanding that all endogenous variables are state-contingent.

3.2 Non-Stochastic Steady State

In Appendix 7.2 we derive all the calculations for the steady state variables. We get the following results for our model:

$$\overline{A} = A$$

$$\overline{\Upsilon} = \exp\left(\frac{1-\phi_2}{2-\phi_1-\phi_2}\ln\Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2}\ln\Upsilon^H\right)$$

$$\overline{r} = \frac{1}{\beta} - 1 + \delta$$

$$\left(\frac{\overline{K}}{\overline{L^w}}\right) = \left(\frac{\alpha A}{\overline{r}}\right)^{\frac{1}{1-\alpha}}$$

$$(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}} = \overline{w}$$

$$\overline{L^c} = \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)$$

$$\overline{\Psi} = \Upsilon\left(\overline{L^c}\right)^{1-\varphi}$$

$$\overline{X} = \delta\overline{K}$$

$$\overline{L^w} = \left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\overline{K}$$

$$\overline{Y} = \frac{1}{\alpha}\overline{r}\overline{K}$$

$$\overline{C} = \left(\frac{\overline{r}}{\alpha} - \delta\right)\overline{K}$$

$$\overline{K} = \frac{\left(1 - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)}{\left(\eta\left(\frac{\overline{r}}{\alpha}-\delta\right) + (1-\tau)\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)\left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\right)}$$

3.3 Log-Linearization

To log-linearize the model we follow the approach in Uhlig (1999). For any variable X we will define the period t log-deviation from its nonstochastic steady-state value:

$$\widetilde{X}_t \equiv \ln X_t - \ln \overline{X}$$

Then:

$$X_t = \overline{X} e^{\widetilde{X}_t}$$

We have the following system of log-linearized equations (find the calculations in Appendix 7.3):

$$\overline{Y}\widetilde{Y}_t = \overline{C}\widetilde{C}_t + \overline{X}\widetilde{X}_t$$

$$\widetilde{Y}_t = \widetilde{A}_t + \alpha\widetilde{K}_t + (1-\alpha)\widetilde{L_t^w}$$

$$\widetilde{\Psi}_t = \widetilde{\Upsilon}_t + (1-\varphi)\widetilde{L_t^c}$$

$$\widetilde{w}_t = \widetilde{Y}_t - \widetilde{L_t^w}$$

$$\widetilde{r}_t = \widetilde{Y}_t - \widetilde{K}_t$$

$$\delta\widetilde{X}_t = \widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t$$

$$\widetilde{C}_t = \widetilde{w}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^c}$$

$$\widetilde{C}_t = \widetilde{\Upsilon}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \left(\varphi + \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\right)\widetilde{L_t^c}$$

$$0 = E_t\left\{\widetilde{C}_{t+1} - \beta\overline{r}\widetilde{r}_{t+1} - \widetilde{C}_t\right\}$$

$$\widetilde{A}_{t+1} = \rho\widetilde{A}_t + \varepsilon_{t+1}$$

Where $\varepsilon_{t+1} \sim iid\ N(0, \sigma_\varepsilon^2)$.

4 Calibration and Results

The calibration we are going to select will be related to what we see in the US time series data and from previous bibliography.

Table 1: Parameter calibration used in Hansen (1985) and own estimates.

Parameter	**Method**	**Value**	**Description**
β	Estimated	0.98	Discount factor
α	Hansen (1985)	0.36	Capital share
$1-\varphi$	Calibrated	0.9	Labor share in the corruption sector.
η	Calibrated	0.5	Relative weight on leisure
δ	Hansen (1985)	0.025	Depreciation rate of capital
τ	Data	0.242	Labor income tax rate
A	Calibration	1	Technology main activity normalized to one.
Υ^H	Calibration	1.2	Technology corruption activity high.
Υ^L	Calibration	0.8	Technology corruption activity low.
ρ	Estimated	0.8	Persistence parameter of the TFP process.
ϕ_1	Calibrated	0.3	Persistence parameter of the low corruption regime.
ϕ_2	Calibrated	0.8	Persistence parameter of the high corruption regime.
σ_ε	Hansen (1985)	0.00712	TFP process st. error

$\alpha = 0.36$ because the time series data for the USA from Kydland and Prescott (1982) indicates that the participation of capital in the product is approximately 36%. $\delta = 0.025$ (quarterly) implies an annual depreciation rate of around 10%. $\beta = 0.98$ implies a steady-state annual interest rate equal to 4.5%. $\eta = 0.5$ implies that in a steady state households will decide to work 1/3 of their time allocation, divided between corruption and main activity labor, which matches the data indicating that agents usually allocate 8 of the 24 hours a day to labor market activities. The employee net average tax rate τ has been taken from the US data

in 2023 according to the US Department of State. We assume that the corruption shock is more volatile than the technology shock. We assume that the labor share in the corruption sector $1-\varphi=0.9$, $\Upsilon^L=0.8$ and $\Upsilon^H=1.2$ in an attempt to match the investment GDP loss estimated in Mauro (1995).

With this calibration, the non-stochastic steady state values of our model are:

Table 2: Data Averages and Long-run Solution

Variable	**Data**	**Model**	**Description**
$\overline{Y}$	N/A	1.11	Steady-state output
$\overline{C}/\overline{Y}$	0.686	0.8018	Consumption-to-output ratio
$\overline{X}/\overline{Y}$	0.201	0.1982	Investment-to-output ratio
$\overline{K}/\overline{Y}$	3.500	7.9281	Capital-to-output ratio
$\overline{\Psi}/\overline{Y}$	≈ 0.05	0.0420	Corruption-to-output ratio
$\overline{w}\overline{L^w}/\overline{Y}$	0.629	0.6400	Labor main activity income-to-output ratio
$\overline{r}\overline{K}/\overline{Y}$	0.359	0.3599	Capital income-to-output ratio
$\overline{L^w}$	0.333	0.3379	Share of total time spent working in the main activity
$\overline{L^c}$	N/A	0.0262	Share of total time spent working in the corruption activity
$\overline{r}$	0.040	0.0454	After-tax net return on capital

The calibration for consumption is the one that seems to be a little far off from the actual data. The reason is because in this model we do not have government spending in our model, the aggregate output is only the sum of consumption and investment. With respect to the rest of the calibrations, like the case of the capital-to-output ratio, we follow Kydland & Prescott (1982). The investment-to-output ratio is the share we see in the actual data. The corruption-to-output ratio is the new share this model presents. It represents the percentage of output that is lost due to corruption. In our model, 4% of the US Output is lost on average every year (This number is obtained from the output's steady state when we equal the corruption shocks to zero, having the same economy but no corruption at all). On the

other hand, we can see that the output, consumption and productive labor falls instantly when we face a corruption shock of 1%. Investment also falls 0.8% approximately when facing a corruption shock. One empirical pattern frequently observed in the data is that countries with high levels of corruption and weak institutions tend to experience significantly lower levels of gross investment. This negative relationship between institutional quality and investment is well documented across various contexts.

Corruption introduces uncertainty, increases transaction costs, and distorts the allocation of resources. When institutions fail to provide credible enforcement of property rights and contracts, the incentives to invest diminish. Firms may become reluctant to engage in long-term capital accumulation, fearing expropriation, bribe extraction, or policy reversals. Moreover, public investment itself is often misallocated or inefficient in highly corrupt environments, further reducing the marginal productivity of private capital.

As a result, economies with fragile institutions and pervasive corruption often display persistently low capital formation, which in turn hampers productivity growth and long-run development.

The impulse-response functions in our calibrated model when $\phi_1 = 0.3$ and $\phi_2 = 0.8$ are:

Figure 1: Impulse-Response Functions to a positive technological innovation of 1% with $\phi_1 = 0.3$ and $\phi_2 = 0.8$.

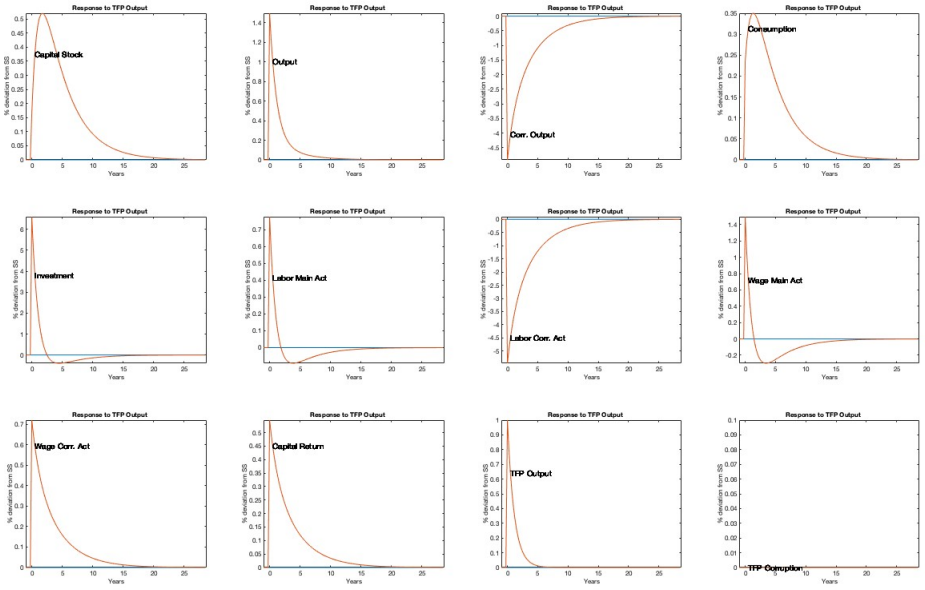


Figure 2: Impulse-Response Functions to a positive corruption innovation of 1% with $\phi_1 = 0.3$ and $\phi_2 = 0.8$.

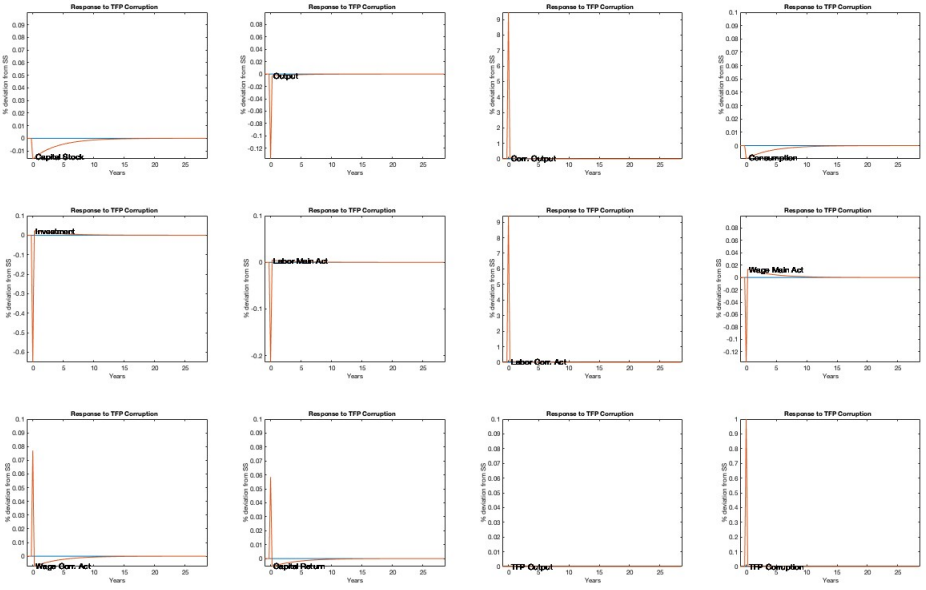
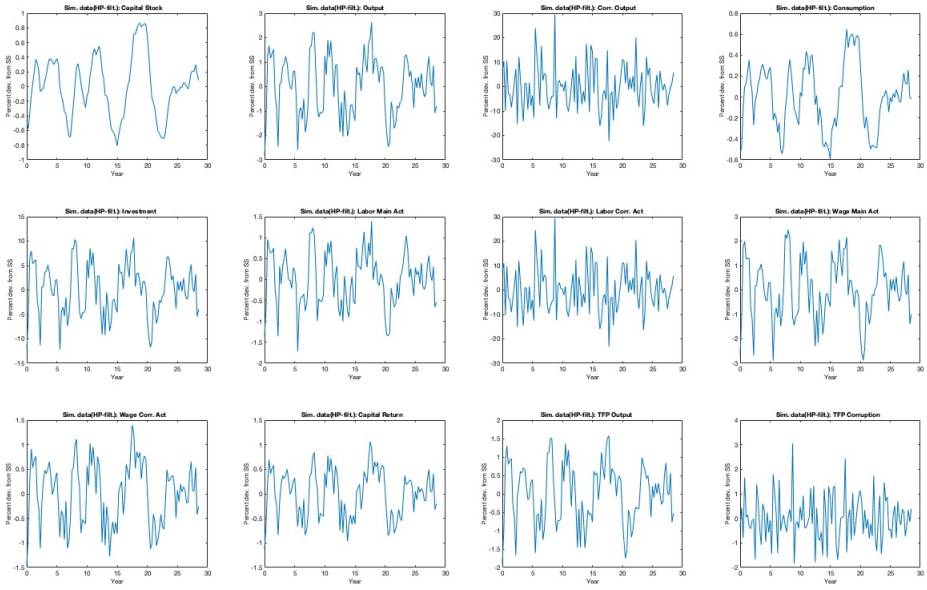
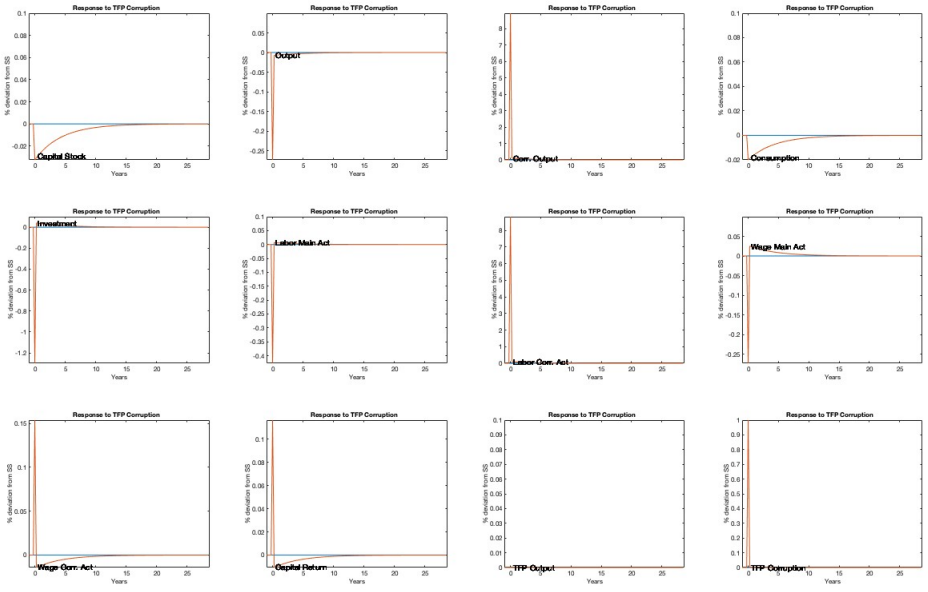


Figure 3: Cyclical component of the variables for the US Data. Hodrick-Prescott (1980) filter with parameter 1600, this is, for quarterly cycles with $\phi_1 = 0.3$ and $\phi_2 = 0.8$.



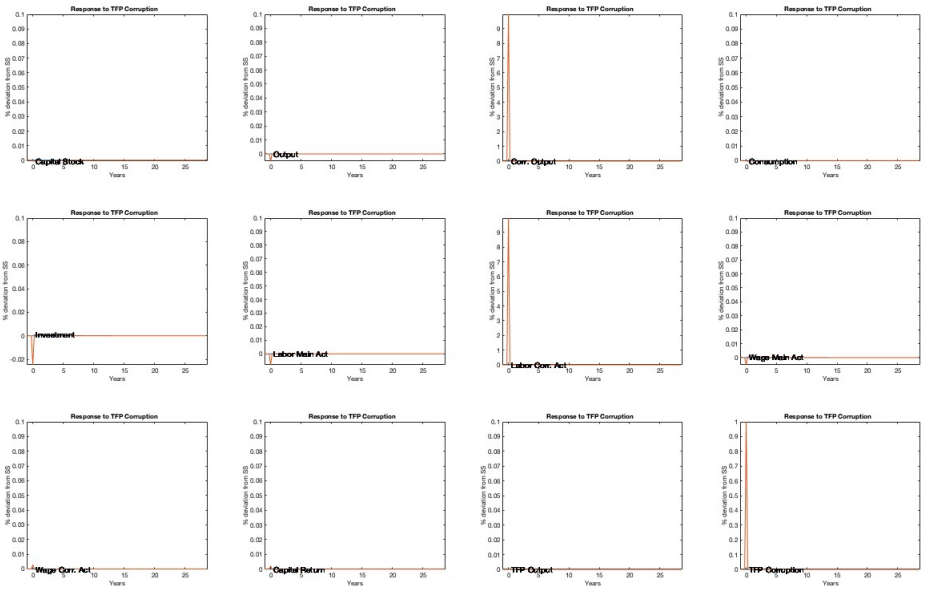
Moreover, when we have a extreme values for $\phi_2 = 1$ (the probability of being next period in a high corruption scenario given that this period corruption is also high) and $\phi_2 = 0$, corruption shock responds in a different way. All variables respond more pronouncedly than before, as corruption becomes a more attractive activity, and our steady state slightly changes, decreasing variables like investment, consumption and output. Corruption will always be present in all possible states of nature. Variables like Consumption, investment and output will fall more significantly than before.

Figure 4: Impulse-Response Functions to a positive corruption innovation of 1% with $\phi_1 = 1$ and $\phi_2 = 0$.



Finally, when we have a extreme values for $\phi_2 = 0$ (the probability of being next period in a high corruption scenario given that this period corruption is also high) and $\phi_2 = 1$, corruption is not an attractive activity anymore thus all possible states of nature will reflect a low corruption in the future. This shock will have virtually no effect compared to the previous one. We will have increasing variables like investment, consumption and output and the corruption shock won't practically affect them.

Figure 5: Impulse-Response Functions to a positive corruption innovation of 1%.



5 Conclusion

In this paper, we developed a novel RBC model to analyze the economic impact of corruption on output, particularly within a steady-state framework. Our model diverges from traditional approaches by treating corruption as a rent-seeking activity, separate from the main productive sector, thus capturing the trade-off agents face between formal labor and corruption activities. By incorporating corruption with its own technology in the economy, we illustrated how this unproductive rent-seeking activity distorts resource allocation.

The model offers significant insights into how corruption-driven policy distortions, especially labor taxes in the formal sector, encourage individuals to shift to corruption activities. This endogenous shift reduces the effective labor force in the productive sector, thereby lowering the economy's overall output. By simulating this scenario, we find that corruption in our model leads to an estimated output loss of approximately 4% of the US GDP, highlighting the substantial economic cost that corruption imposes.

Our framework allows for further exploration of policy interventions aimed at reducing the incentive for corruption. One implication of our findings is that lowering labor taxes in the main productive sector could help deter labor shifts toward corruption. Additionally, a more stringent control of unobserved economic activities could mitigate these losses by reducing the profitability of corruption. This study highlights the urgent need for policies that address the systemic incentives that promote corruption, as even small improvements in governance could yield considerable economic benefits.

In conclusion, the model demonstrates the hidden yet powerful impact of corruption on an economy's steady state. The estimated GDP loss due to corruption in the US signals the importance of combating these activities to enhance productivity and achieve better resource allocation. The findings contribute to a deeper understanding of corruption's cost in a cyclical rather than in a growth scenario, which policymakers should consider to foster more resilient and efficient economies.

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7 Appendix

7.1

In this appendix, we will prove that with the separable utility function we've chosen, the household problem for the i agents can be solved as a representative agent's utility-maximization problem:

$$U\left(C_t^i, h_t^i\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t^i + \eta \ln h_t^i\right]$$

The FOCs are:

$$\begin{aligned}
\{C_t^i\} \quad & \lambda_t^i = \frac{1}{C_t^i} \\
\{(L_t^i)^w\} \quad & \frac{\eta}{1-(L_t^i)^w-(L_t^i)^c} = \lambda_t^i(1-\tau)w_t \\
\{(L_t^i)^c\} \quad & \frac{\eta}{1-(L_t^i)^w-(L_t^i)^c} = \lambda_t^i \Upsilon_t^i \left(1-\varphi^i\right)\left((L_t^i)^c\right)^{-\varphi^i} \\
\{K_{t+1}^i\} \quad & \lambda_t^i = E_t\left(\beta\lambda_{t+1}^i(1-\delta+r_{t+1})\right) \\
\{\lambda_t\} \quad & C_t^i + X_t^i = r_t K_t^i + (1-\tau)(L_t^i)^w w_t + \Upsilon_t \left(L_t^c\right)^{1-\varphi} \\
\{TC_K^i\} \quad & \lim_{t\to\infty} \beta^t \lambda_t^i K_{t+1}^i = 0
\end{aligned}$$

Taking the FOC for consumption $\{C_t^i\}$ for $j \neq i$ and dividing them we get:

$$\frac{C_t^i}{C_t^j} = \frac{\lambda^i}{\lambda^j}$$

$$C_t^i = \left(\frac{\lambda^i}{\lambda^j}\right) C_t^j$$

Adding for all i:

$$\underbrace{\sum_{i=1}^{I} C_t^i}_{=C_t} = (\lambda^j) C_t^j \sum_{i=1}^{I} \left(\frac{1}{\lambda^i}\right)$$

$$C_t = (\lambda^j) C_t^j \sum_{i=1}^{I} \left(\frac{1}{\lambda^i}\right)$$

Clearing C_t^j:

$$C_t^j = \left(\frac{1}{\lambda^j}\right) C_t \frac{1}{\sum_{i=1}^{I} \left(\frac{1}{\lambda^i}\right)}$$

Now adding for all j:

$$\underbrace{\sum_{j=1}^{J} C_t^j}_{=C_t} = C_t \frac{1}{\sum_{i=1}^{I} \left(\frac{1}{\lambda^i}\right)} \sum_{i=1}^{J} \left(\frac{1}{\lambda^j}\right)$$

Since the sum for all j and i of the lambdas are equal, we can simplify them and we are left with:

$$C_t = C_t \quad \square$$

We see that it is the same aggregate as in an economy with a single agent.

7.2

In this appendix, we are going to derive the ergodic steady state equations.

We use upper bars to denote nonstochastic steady-state values. Setting $\varepsilon_{t+1} = 0$ and $A_{t+1} = A_t$

into $\ln(A_{t+1}) = (1-\rho)\ln(A) + \rho \ln A_t + \varepsilon_{t+1}$ we get

$$\overline{A} = A$$

The ergodic steady-state equilibrium in a regime-switching DSGE model is computed by solving for the values of endogenous variables that are constant across time but conditional on the persistence of the underlying Markov state. As shown in Farmer, Waggoner and Zha (2011) and applied in Bianchi (2013), the model's endogenous variables are defined for each regime: typically high and low productivity or corruption states, and the solution must satisfy the system of expectational equations conditional on each regime. The stationary equilibrium is obtained by setting all stochastic shocks to their unconditional means and solving for the variables such that they remain constant within each regime. This yields a system of nonlinear equations in which agents' expectations incorporate the transition probabilities across regimes. The final ergodic distribution is derived from the invariant distribution of the Markov process and the policy functions evaluated at the steady state of each regime.

We have defined before the Markov transition matrix for the corruption regime to be given by:

$$\mathcal{P} = \begin{bmatrix} \phi_1 & 1-\phi_2 \\ 1-\phi_1 & \phi_2 \end{bmatrix}$$

where ϕ_1 is the probability of staying in the low-corruption regime and ϕ_2 the probability of staying in the high-corruption regime. The ergodic distribution $\pi = (\pi_L, \pi_H)$ solves $\pi \cdot \mathcal{P} = \pi$ with $\pi_L + \pi_H = 1$. Solving this system yields:

$$\pi_L = \frac{1-\phi_2}{2-\phi_1-\phi_2}, \quad \pi_H = \frac{1-\phi_1}{2-\phi_1-\phi_2}$$

Thus, the ergodic expectation of log corruption productivity is given by:

$$\ln \overline{\Upsilon} = E[\ln \Upsilon] = \pi_L \ln \Upsilon^L + \pi_H \ln \Upsilon^H = \frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H$$

$$\Rightarrow \overline{\Upsilon} = \exp\left(\ln \overline{\Upsilon}\right)$$

This expression is used in the steady-state derivation to replace the constant corruption productivity Υ with its ergodic mean $\overline{\Upsilon}$, consistent with a Markov-switching model solved under rational expectations. Following Schorfheide (2005), we compute the ergodic steady-state value of $\ln \Upsilon$ by taking the expectation across regimes using the stationary distribution of the Markov process. Then, it is defined as:

$$\overline{\Upsilon} = \exp\left(\frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H\right)$$

This average value captures the long-run expected productivity of the corruption technology across regimes and replaces the fixed parameter Υ in all subsequent steady state derivations.

Now, we want to observe how the ergodic mean of the steady state corruption productivity depends on ϕ_1 or the probability of being in a low corruption state the next period given that this period corruption is low and how this mean also depends on ϕ_2 that is the probability of remaining the next period with high corruption given that this period corruption is also high.

We now derive how $\overline{\Upsilon}$ changes with the transition probabilities ϕ_1 and ϕ_2 of the two-state Markov chain. Denote:

$$f(\phi_1, \phi_2) = \frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H \Rightarrow \overline{\Upsilon} = \exp(f(\phi_1, \phi_2))$$

$$\frac{\partial \overline{\Upsilon}}{\partial \phi_l} = \exp(f(\phi_1, \phi_2)) \cdot \frac{\partial f(\phi_1, \phi_2)}{\partial \phi_l} \quad \forall l \in [1, 2]$$

As the exponential term is always strictly positive, the sign of the derivative $\frac{\partial \overline{\Upsilon}}{\partial \phi_l}$ depends solely on the sign of $\frac{\partial f(\phi_1,\phi_2)}{\partial \phi_l}$.

Let:

$$f(\phi_1, \phi_2) = \frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H$$

We compute:

$$\frac{\partial f}{\partial \phi_1} = \frac{\partial}{\partial \phi_1} \left[\frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H \right]$$

Let us differentiate each term:

$$\frac{\partial}{\partial \phi_1} \left(\frac{1-\phi_2}{2-\phi_1-\phi_2} \right) = \frac{(1-\phi_2)(1)}{(2-\phi_1-\phi_2)^2} = \frac{1-\phi_2}{(2-\phi_1-\phi_2)^2}$$

$$\frac{\partial}{\partial \phi_1} \left(\frac{1-\phi_1}{2-\phi_1-\phi_2} \right) = \frac{-(2-\phi_1-\phi_2)+(1-\phi_1)}{(2-\phi_1-\phi_2)^2} = -\frac{(1-\phi_2)}{(2-\phi_1-\phi_2)^2}$$

Therefore:

$$\frac{\partial f}{\partial \phi_1} = \frac{1-\phi_2}{(2-\phi_1-\phi_2)^2} \ln \Upsilon^L - \frac{1-\phi_2}{(2-\phi_1-\phi_2)^2} \ln \Upsilon^H$$

$$\frac{\partial f}{\partial \phi_1} = \underbrace{\frac{1-\phi_2}{(2-\phi_1-\phi_2)^2}}_{\geq 0} \cdot \underbrace{\left[\ln \Upsilon^L - \ln \Upsilon^H\right]}_{\leq 0}$$

$$\frac{\partial f}{\partial \phi_1} \leq 0$$

Similarly, for ϕ_2:

$$\frac{\partial f}{\partial \phi_2} = \frac{\partial}{\partial \phi_2} \left[\frac{1-\phi_2}{2-\phi_1-\phi_2} \ln \Upsilon^L + \frac{1-\phi_1}{2-\phi_1-\phi_2} \ln \Upsilon^H \right]$$

$$\frac{\partial}{\partial\phi_2}\left(\frac{1-\phi_2}{2-\phi_1-\phi_2}\right)=\frac{-(2-\phi_1-\phi_2)+(1-\phi_2)}{(2-\phi_1-\phi_2)^2}=-\frac{1-\phi_1}{(2-\phi_1-\phi_2)^2}$$

$$\frac{\partial}{\partial\phi_2}\left(\frac{1-\phi_1}{2-\phi_1-\phi_2}\right)=\frac{1-\phi_1}{(2-\phi_1-\phi_2)^2}$$

Hence:

$$\frac{\partial f}{\partial\phi_2}=-\frac{1-\phi_1}{(2-\phi_1-\phi_2)^2}\ln\Upsilon^L+\frac{1-\phi_1}{(2-\phi_1-\phi_2)^2}\ln\Upsilon^H$$

$$\frac{\partial f}{\partial\phi_2}=\underbrace{\frac{1-\phi_2}{(2-\phi_1-\phi_2)^2}}_{\geq 0}\cdot\underbrace{\left[-\ln\Upsilon^L+\ln\Upsilon^H\right]}_{\geq 0}$$

$$\frac{\partial f}{\partial\phi_2}\geq 0$$

Setting $C_t=C_{t+1}=\overline{C}$ and $r_{t+1}=\overline{r}$ into $E_t\left\{\beta\left(\frac{C_t}{C_{t+1}}\right)(r_{t+1}+1-\delta)\right\}=1$ we get

$$\overline{r}=\frac{1}{\beta}-1+\delta$$

$$\overline{r}=\rho+\delta$$

We know that $\alpha A_t\left(\frac{K_t}{L_t^w}\right)^{\alpha-1}=r_t$. So, given that we have set $r_t=\overline{r}$ and $A_t=\overline{A}$, we get:

$$\left(\frac{\overline{K}}{\overline{L^w}}\right)=\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{1}{1-\alpha}}$$

We also know that $(1-\alpha)A_t\left(\frac{K_t}{L_t^w}\right)^{\alpha}=w_t$. Using the expression found above for $\left(\frac{\overline{K}}{\overline{L^w}}\right)$:

$$(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}=\overline{w}$$

Moreover, from the Corruption production function (recalling that we have set $\Upsilon_t = \Upsilon$), due to the market clearing condition in the labor market, we know that:

$$(1-\varphi)\Upsilon(\overline{L^c})^{-\varphi} = (1-\tau)\overline{w}$$

$$(1-\varphi)\Upsilon(\overline{L^c})^{-\varphi} = (1-\tau)\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

Working a little with the previous expression we get:

$$(\overline{L^c})^{-\varphi} = \frac{(1-\tau)}{(1-\varphi)\Upsilon}\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

$$\overline{L^c} = \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)$$

From the production function of the corruption activity, we have that $\Psi_t = \Upsilon_t \left(L_t^c\right)^{1-\varphi}$. So, replacing the steady state variable for labor in the corruption function found before, we have the corruption production:

$$\overline{\Psi} = \Upsilon\left(\overline{L^c}\right)^{1-\varphi}$$

$$\overline{\Psi} = \Upsilon\left(\left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)^{1-\varphi}$$

Now, setting $K_{t+1} = K_t = \overline{K}$ into the capital accumulation condition ($X_t = K_{t+1} - (1-\delta)K_t$), we get:

$$\overline{X} = \delta\overline{K}$$

From $\left(\frac{\overline{K}}{\overline{L^w}}\right) = \left(\frac{\alpha A}{\overline{r}}\right)^{\frac{1}{1-\alpha}}$, we get:

$$\overline{L^w} = \left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\overline{K}$$

From the production function of the main activity, we have that $\overline{Y} = A\overline{K}^{\alpha}\left(\overline{L^w}\right)^{1-\alpha}$, so

replacing the expression for $\overline{L^w}$ above we get:

$$\overline{Y} = \frac{1}{\alpha}\overline{r}\overline{K}$$

From the Aggregated Consistency in the market of the consumption good $Y_t = C_t + X_t$, we have that:

$$\overline{C} = \overline{Y} - \overline{X}$$

$$\overline{C} = \frac{1}{\alpha}\overline{r}\overline{K} - \delta\overline{K}$$

$$\overline{C} = \left(\frac{\overline{r}}{\alpha} - \delta\right)\overline{K}$$

So, taking the first order condition from the labor choice of the main activity: $\frac{\eta}{1-L_t^w-L_t^c} = \frac{1}{C_t}(1-\tau)w_t$. Re-arranging and taking the steady state, $\eta\overline{C} = \left(1-\overline{L^w}-\overline{L^c}\right)(1-\tau)w_t$. Now, replacing the expressions found before for $\overline{L^c}$, $\overline{L^w}$, $\overline{C}$ and $\overline{w^c}$, we can clear the expression for the state variable $\overline{K}$ and solve our model:

$$\eta\left(\frac{\overline{r}}{\alpha} - \delta\right)\overline{K} = \left(1 - \left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\overline{K} - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\overline{w}$$

$$\eta\left(\frac{\overline{r}}{\alpha} - \delta\right)\overline{K} + (1-\tau)\overline{w}\left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\overline{K} = \left(1 - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\overline{w}$$

$$\left(\eta\left(\frac{\overline{r}}{\alpha} - \delta\right) + (1-\tau)\overline{w}\left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\right)\overline{K} = \left(1 - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\overline{w}$$

$$\overline{K} = \frac{\left(1 - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}}\left(A^{-\frac{1}{(1-\alpha)\varphi}}\left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\overline{w}}{\left(\eta\left(\frac{\overline{r}}{\alpha} - \delta\right) + (1-\tau)\overline{w}\left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\right)}$$

If we replace for the value of the salary of the main activity in the capital equation, we get:

$$\overline{K} = \frac{\left(1 - \left(\frac{(1-\varphi)\Upsilon}{(1-\tau)(1-\alpha)}\right)^{\frac{1}{\varphi}} \left(A^{-\frac{1}{(1-\alpha)\varphi}} \left(\frac{\alpha A}{\overline{r}}\right)^{-\frac{\alpha}{(1-\alpha)\varphi}}\right)\right)(1-\tau)\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)}{\left(\eta\left(\frac{\overline{r}}{\alpha} - \delta\right) + (1-\tau)\left((1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha A}{\overline{r}}\right)^{\frac{\alpha}{1-\alpha}}\right)\left(\frac{\overline{r}}{\alpha A}\right)^{\frac{1}{1-\alpha}}\right)} \quad \square$$

7.3

In this appendix, we are going to derive the log-linearized equations of the model.

We will approximate functions $(\widetilde{X}_t, \widetilde{Y}_t)$ around (0,0). This will give us linear functions of $\widetilde{X}_t$ and $\widetilde{Y}_t$. We have to remember the following properties we are not going to prove. 1) $e^{\widetilde{X}_t + a\widetilde{Y}_t} \approx 1 + \widetilde{X}_t + a\widetilde{Y}_t$. Corollary: $e^{\widetilde{X}_t} \approx 1 + \widetilde{X}_t$, 2) $\widetilde{X}_t\widetilde{Y}_t \approx 0$ and 3) $E_t\left\{ae^{\widetilde{X}_{t+1}}\right\} \cong a + aE_t\left\{\widetilde{X}_{t+1}\right\}$

Now we log-linearize the 12 equations that characterize the equilibrium of our model.

For TFP we get

$$\begin{aligned}
\widetilde{A}_{t+1} &\equiv \ln A_{t+1} - \ln \overline{A} \\
&= (1-\rho)\ln A + \rho \ln A_t + \varepsilon_{t+1} - \ln \overline{A} \\
&= (1-\rho)\ln \overline{A} + \rho \ln A_t + \varepsilon_{t+1} - \ln \overline{A} \\
&= \rho\left(\ln A_t - \ln \overline{A}\right) + \varepsilon_{t+1}
\end{aligned}$$

Then

$$\widetilde{A}_{t+1} = \rho\widetilde{A}_t + \varepsilon_{t+1}$$

For output we get:

$$\begin{aligned}
\widetilde{Y}_t &\equiv \ln Y_t - \ln \overline{Y} \\
&= \ln\left(A_t K_t^{\alpha}(L_t^w)^{1-\alpha}\right) - \ln\left(\overline{AK}^{\alpha}\overline{L^w}^{1-\alpha}\right) \\
&= \ln A_t + \alpha \ln K_t + (1-\alpha)\ln L_t^w - \ln \overline{A} - \alpha \ln \overline{K} - (1-\alpha)\ln \overline{L^w} \\
&= \left(\ln A_t - \ln \overline{A}\right) + \alpha\left(\ln K_t - \ln \overline{K}\right) + (1-\alpha)\left(\ln L_t^w - \ln \overline{L^w}\right)
\end{aligned}$$

Then

$$\widetilde{Y}_t = \widetilde{A}_t + \alpha \widetilde{K}_t + (1-\alpha)\widetilde{L_t^w}$$

From the corruption production function we have:

$$\begin{aligned}
\widetilde{\Psi}_t \equiv \ln \Psi_t - \ln \overline{\Psi} & \\
&= \ln(\Upsilon_t \, (L_t^c)^{1-\varphi}) - \ln(\overline{\Upsilon L^c}^{1-\varphi}) \\
&= \ln \Upsilon_t + (1-\varphi) \ln L_t^c - \ln \overline{\Upsilon} - (1-\varphi) \ln \overline{L^c} \\
&= (\ln \Upsilon_t - \ln \overline{\Upsilon}) + (1-\varphi)(\ln L_t^c - \ln \overline{L^c})
\end{aligned}$$

Then:

$$\widetilde{\Psi}_t = \widetilde{\Upsilon}_t + (1-\varphi)\widetilde{L_t^c}$$

From $w_t = (1-\alpha)\frac{Y_t}{L_t^w}$ we get

$$\begin{aligned}
\widetilde{w_t} \equiv \ln w_t - \ln \overline{w} & \\
&= \ln\left((1-\alpha)\frac{Y_t}{L_t^w}\right) - \ln\left((1-\alpha)\frac{\overline{Y}}{\overline{L^w}}\right) \\
&= \ln(1-\alpha) + \ln Y_t - \ln L_t^w - \ln(1-\alpha) - \ln \overline{Y} + \ln \overline{L^w} \\
&= \left(\ln Y_t - \ln \overline{Y}\right) - \left(\ln L_t^w - \ln \overline{L^w}\right)
\end{aligned}$$

Then

$$\widetilde{w_t} = \widetilde{Y_t} - \widetilde{L_t^w}$$

Analogously, from $\frac{\eta}{1-L_t^w-L_t^c} = \frac{1}{C_t}(1-\varphi)\frac{\Psi_t}{L_t^c}$, we get:

$$\widetilde{w_t} = \widetilde{\Psi_t} - \widetilde{L_t^c}$$

Notice that all the previous expressions are exact relations, no approximations were needed to

derive them. For the following log-linearizations we use the approximations derived previously. For simplicity, we use the = sign instead of $\approx$.

From $Y_t = C_t + X_t$ we get

$$\begin{aligned}
\overline{Y}e^{\widetilde{Y}_t} &= \overline{C}e^{\widetilde{C}_t} + \overline{X}_e^{\widetilde{X}_t} \\
\overline{Y}\left(1+\widetilde{Y}_t\right) &= \overline{C}\left(1+\widetilde{C}_t\right) + \overline{X}\left(1+\widetilde{X}_t\right) \\
\overline{Y} + \overline{Y}\widetilde{Y}_t &= \overline{C} + \overline{C}\widetilde{C}_t + \overline{X} + \overline{X}\widetilde{X}_t \\
\overline{Y} + \overline{Y}\widetilde{Y}_t &= (\overline{C} + \overline{X}) + \overline{C}\widetilde{C}_t + \overline{X}\widetilde{X}_t \\
\overline{Y} + \overline{Y}\widetilde{Y}_t &= \overline{Y} + \overline{C}\widetilde{C}_t + \overline{X}\widetilde{X}_t
\end{aligned}$$

Then

$$\overline{Y}\widetilde{Y}_t = \overline{C}\widetilde{C}_t + \overline{X}\widetilde{X}_t$$

From $X_t = K_{t+1} - (1-\delta)K_t$ we get

$$\begin{aligned}
\overline{X}e^{\widetilde{X}_t} &= \overline{K}e^{\widetilde{K}_{t+1}} - (1-\delta)\overline{K}e^{\widetilde{K}_t} \\
\overline{X}\left(1+\widetilde{X}_t\right) &= \overline{K}\left(1+\widetilde{K}_{t+1}\right) - (1-\delta)\overline{K}\left(1+\widetilde{K}_t\right) \\
\overline{X} + \overline{X}\widetilde{X}_t &= \overline{K} + \overline{K}\widetilde{K}_{t+1} - (1-\delta)\overline{K} - (1-\delta)\overline{K}\widetilde{K}_t \\
\overline{X} + \overline{X}\widetilde{X}_t &= \delta\overline{K} + \overline{K}\left[\widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t\right] \\
\overline{X} + \overline{X}\widetilde{X}_t &= \overline{X} + \overline{K}\left[\widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t\right] \\
\overline{X}\widetilde{X}_t &= \overline{K}\left[\widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t\right] \\
\delta\overline{K}\widetilde{X}_t &= \overline{K}\left[\widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t\right]
\end{aligned}$$

Then

$$\delta\widetilde{X}_t = \widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t$$

From $\frac{\eta}{1-L_t^w-L_t^c} = \frac{1}{C_t}(1-\tau)w_t$, we have:

$$\eta C_t = (1 - L_t^w - L_t^c)(1-\tau)w_t$$

$$\eta\overline{C}e^{\widetilde{C}_t} = (1 - \overline{L^w}e^{\widetilde{L_t^w}} - \overline{L^c}e^{\widetilde{L_t^c}})(1-\tau)\overline{w}e^{\widetilde{w_t}}$$

$$\eta\overline{C}e^{\widetilde{C}_t} = (1 - \overline{L^w}e^{\widetilde{L_t^w}} - \overline{L^c}e^{\widetilde{L_t^c}})(1-\tau)\overline{w}e^{\widetilde{w_t}}$$

$$\eta\overline{C}\left(1+\widetilde{C}_t\right) = \left[1 - \overline{L^w}\left(1+\widetilde{L_t^w}\right) - \overline{L^c}\left(1+\widetilde{L_t^c}\right)\right]\overline{w}\left(1+\widetilde{w}_t\right)(1-\tau)$$

$$\eta\overline{C} + \eta\overline{C}\widetilde{C}_t = \left[(1 - \overline{L^w} - \overline{L^c}) - \overline{L^w}\widetilde{L_t^w} - \overline{L^c}\widetilde{L_t^c}\right](\overline{w} + \overline{w}\widetilde{w}_t)\underbrace{(1-\tau)}_{\Theta}$$

$$\eta\overline{C}+\eta\overline{C}\widetilde{C}_t = \overline{w}(1-\overline{L^w}-\overline{L^c})\Theta+\overline{w}\widetilde{w}_t(1-\overline{L^w}-\overline{L^c})\Theta-\overline{w}\overline{L^w}\widetilde{L_t^w}\Theta-\overline{w}\widetilde{w}_t\overline{L^w}\widetilde{L_t^w}\Theta-\overline{w}\overline{L^c}\widetilde{L_t^c}\Theta-\overline{w}\widetilde{w}_t\overline{L^c}\widetilde{L_t^c}\Theta$$

Now, recalling that, from the agent's FOC, $\eta\overline{C} = \overline{w}(1 - \overline{L^w} - \overline{L^c})\Theta$:

$$\overline{w}(1-\overline{L^w}-\overline{L^c})\Theta+(1-\overline{L^w}-\overline{L^c})\overline{w}\Theta\widetilde{C}_t = \overline{w}(1-\overline{L^w}-\overline{L^c})\Theta+\overline{w}\widetilde{w_t}(1-\overline{L^w}-\overline{L^c})\Theta-\overline{w}\overline{L^w}\widetilde{L_t^w}\Theta-\underbrace{\overline{w}\overline{L^w}\widetilde{w_t}\widetilde{L_t^w}}_{\approx 0}\Theta-\overline{w}\overline{L^c}\widetilde{L_t^c}\Theta-\underbrace{\overline{w}\overline{L^c}\widetilde{w_t}\widetilde{L_t^c}}_{\approx 0}\Theta$$

$$(1 - \overline{L^w} - \overline{L^c})\overline{w}\Theta\widetilde{C}_t = \overline{w}\widetilde{w_t}(1 - \overline{L^w} - \overline{L^c})\Theta - \overline{w}\overline{L^w}\widetilde{L_t^w}\Theta - \overline{w}\overline{L^c}\widetilde{L_t^c}\Theta$$

$$\widetilde{C}_t = \widetilde{w_t} - \frac{\overline{L^w}\widetilde{L_t^w}}{1 - \overline{L^w} - \overline{L^c}} - \frac{\overline{L^c}\widetilde{L_t^c}}{1 - \overline{L^w} - \overline{L^c}}$$

We have an expression for consumption.

We will also have the expression for consumption depending on the corruption function's TFP. We start from the agent's first-order condition:

$$(1-\tau)w_t = \Upsilon_t(1-\varphi)\left(L_t^c\right)^{-\varphi}$$

We take logs on both sides:

$$\ln\left((1-\tau)w_t\right) = \ln\left(\Upsilon_t(1-\varphi)\left(L_t^c\right)^{-\varphi}\right)$$

$$\ln(1-\tau) + \ln w_t = \ln(1-\varphi) + \ln \Upsilon_t - \varphi \ln L_t^c$$

$$\ln w_t = \ln(1-\varphi) - \ln(1-\tau) + \ln \Upsilon_t - \varphi \ln L_t^c$$

Now, we subtract steady state values and define log deviations.[2]:

$$\ln w_t - \ln \overline{w} = (\ln \Upsilon_t - \ln \overline{\Upsilon}) - \varphi(\ln L_t^c - \ln \overline{L^c})$$

$$\widetilde{w}_t = \widetilde{\Upsilon}_t - \varphi \widetilde{L_t^c}$$

Recall the previous log-linearized expression for consumption:

$$\widetilde{C}_t = \widetilde{w}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^c}$$

Now, we replace $\widetilde{w}_t$:

$$\widetilde{C}_t = \left(\widetilde{\Upsilon}_t - \varphi \widetilde{L_t^c}\right) - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^c}$$

$$\widetilde{C}_t = \widetilde{\Upsilon}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \left(\varphi + \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\right)\widetilde{L_t^c}$$

[2]The constants $\ln(1-\varphi)$ and $\ln(1-\tau)$ cancel out because they do not vary over time. They are already embedded in the steady-state wage $\overline{w}$.

From the Euler equation, $1 = E_t\left\{\beta\frac{C_t}{C_{t+1}}\left(r_{t+1}+1-\delta\right)\right\}$ we get

$$
\begin{aligned}
1 &= E_t\left\{\beta\frac{\overline{C}e^{\tilde{C}_t}}{\overline{C}e^{\tilde{C}_{t+1}}}\left(\overline{r}e^{\tilde{r}_{t+1}}+1-\delta\right)\right\}\\
1 &= E_t\left\{\beta e^{\tilde{C}_t-\tilde{C}_{t+1}}\left[\overline{r}\left(1+\widetilde{r}_{t+1}\right)+1-\delta\right]\right\}\\
1 &= E_t\left\{\beta\left(1+\widetilde{C}_t-\widetilde{C}_{t+1}\right)\left[\overline{r}+\overline{r}\widetilde{r}_{t+1}+1-\delta\right]\right\}\\
1 &= E_t\left\{\beta\left(1+\widetilde{C}_t-\widetilde{C}_{t+1}\right)\left[\left(\overline{r}+1-\delta\right)+\overline{r}\widetilde{r}_{t+1}\right]\right\}\\
1 &= E_t\left\{\left(\beta+\beta\widetilde{C}_t-\beta\widetilde{C}_{t+1}\right)\left(\frac{1}{\beta}+\overline{r}\widetilde{r}_{t+1}\right)\right\}\\
1 &= E_t\left\{\beta\left(\frac{1}{\beta}+\overline{r}\widetilde{r}_{t+1}\right)+\beta\widetilde{C}_t\left(\frac{1}{\beta}+\overline{r}\widetilde{r}_{t+1}\right)-\beta\widetilde{C}_{t+1}\left(\frac{1}{\beta}+\overline{r}\widetilde{r}_{t+1}\right)\right\}\\
1 &= E_t\left\{1+\beta\overline{r}\widetilde{r}_{t+1}+\widetilde{C}_t+\beta\overline{r}\widetilde{C}_t\widetilde{r}_{t+1}-\widetilde{C}_{t+1}-\beta\overline{r}\widetilde{C}_{t+1}\widetilde{r}_{t+1}\right\}\\
0 &= E_t\left\{\beta\overline{r}\widetilde{r}_{t+1}+\widetilde{C}_t-\widetilde{C}_{t+1}\right\}
\end{aligned}
$$

Then:

$$0 = E_t\left\{\widetilde{C}_{t+1}-\beta\overline{r}\widetilde{r}_{t+1}-\widetilde{C}_t\right\}$$

So, summarizing, we have the following system of log-linearized equations:

$$\overline{Y}\widetilde{Y}_t = \overline{C}\widetilde{C}_t + \overline{X}\widetilde{X}_t$$

$$\widetilde{Y}_t = \widetilde{A}_t + \alpha\widetilde{K}_t + (1-\alpha)\widetilde{L_t^w}$$

$$\widetilde{\Psi}_t = \widetilde{\Upsilon}_t + (1-\varphi)\widetilde{L_t^c}$$

$$\widetilde{w}_t = \widetilde{Y}_t - \widetilde{L_t^w}$$

$$\widetilde{r}_t = \widetilde{Y}_t - \widetilde{K}_t$$

$$\delta\widetilde{X}_t = \widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t$$

$$\widetilde{C}_t = \widetilde{w}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^c}$$

$$\widetilde{C}_t = \widetilde{\Upsilon}_t - \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}}\widetilde{L_t^w} - \left(\varphi + \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\right)\widetilde{L_t^c}$$

$$0 = E_t\left\{\widetilde{C}_{t+1} - \beta\overline{r}\widetilde{r}_{t+1} - \widetilde{C}_t\right\}$$

$$\widetilde{A}_{t+1} = \rho\widetilde{A}_t + \varepsilon_{t+1}$$

Where $\varepsilon_{t+1} \sim iid\ N(0, \sigma_\varepsilon^2)$ and $\varsigma_{t+1} \sim iid\ N\left(0, \sigma_\varsigma^2\right)$ are independent of each other.

We will define the following column vectors:

$$x_t \equiv \left[\widetilde{K}_{t+1}\right] \quad y_t \equiv \begin{bmatrix} \widetilde{Y}_t \\ \widetilde{\Psi}_t \\ \widetilde{C}_t \\ \widetilde{X}_t \\ \widetilde{L_t^w} \\ \widetilde{L_t^c} \\ \widetilde{r}_t \\ \widetilde{w}_t \end{bmatrix} \quad z_t \equiv \begin{bmatrix} \widetilde{A}_t \\ \widetilde{\Upsilon}_t \end{bmatrix} \quad \epsilon_{t+1} \equiv \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

Then, we can rewrite our log-linearized system in matrix form, as follows:

$$\mathbf{0} = Ax_t + Bx_{t-1} + Cy_t + Dz_t$$

$$\mathbf{0} = E_t \left\{ Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t \right\}$$

$$z_{t+1} = Nz_t + \epsilon_{t+1} \quad \text{with } E_t \left\{ \epsilon_{t+1} \right\} = \mathbf{0}$$

Note that the second equation corresponds solely to the stochastic Euler equation; the first equation corresponds to the following 8 equations of the system; and, finally, the third equation represents the AR(1) process for TFP, which is the last equation of our log-linearized system.

Matrices A, B, C, and D are the following:

$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -\alpha \\ 0 \\ 0 \\ -1 \\ -(1-\delta) \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} \overline{Y} & 0 & -\overline{C} & -\overline{X} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -(1-\alpha) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -(1-\varphi) & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}} & \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}} & 0 & -1 \\ 0 & 0 & 1 & 0 & \frac{\overline{L^w}}{1-\overline{L^w}-\overline{L^c}} & \left(\varphi + \frac{\overline{L^c}}{1-\overline{L^w}-\overline{L^c}}\right) & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}$$

Matrices F, G, H, J, K, L, M are the following:

$$F = [0] \quad G = [0] \quad H = [0]$$

$$J = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -\beta\overline{r} & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Matrix N is:

$$N = \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix}$$

With this matrices, we are now able to solve the model through the approach followed by Uhlig (1999). Matlab is the program used to write the algorithm. □